Multi-level Moderation & Mediation

Kenneth Law (罗胜强)
Department of Management
The Chinese University of Hong Kong
香港中文大学管理系

1

Interaction and Mediation

- Multi-level Moderation
- Multi-level Mediation
- Moderated Mediation
- Mediated Moderation
- Multi-level Moderated Mediation
- Multi-level Mediated Moderation
- Curvilinear Mediation
- Curvilinear Moderated Mediation

2

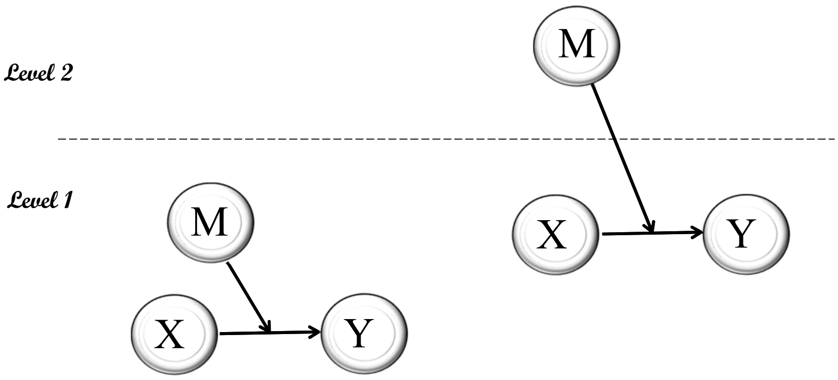
管理学问卷调查
研究方法

罗胜强　姜嬿 著

重庆大学出版社

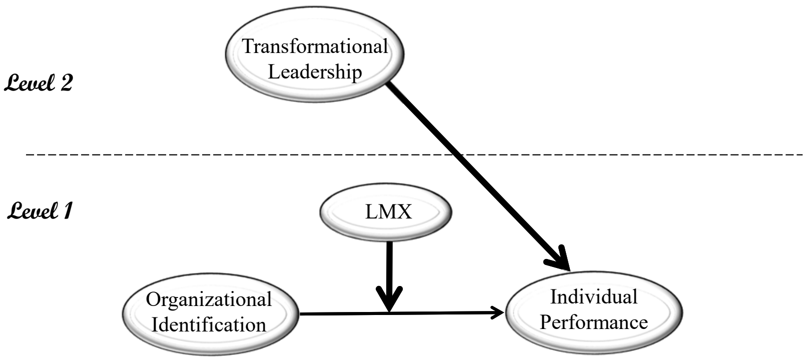
3

Multi-level Moderation



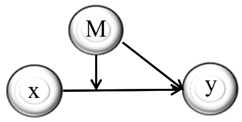
4

Within-level Moderation



Within-level Moderation

```
TITLE:      Sample multi-level program on helping
DATA:       FILE = data.txt;
VARIABLE:   NAMES = group x M xM W;
            USEVARIABLES = help;
            WITHIN = x M xM ;
            BETWEEN =     ;
            CLUSTER = group;
ANALYSIS:   TYPE = TWOLEVEL RANDOM;
MODEL:
        %WITHIN%
          y on x M xM;
          x M xM;
          y ! you can skip this line
        %BETWEEN%
          y;   ! you can skip this line
OUTPUT:     TECH1;
```



$$y_{ij} = \beta_{0j} + \beta_{1j}x_{ij} + \beta_{2j}M_{ij} + \beta_{3j}xM_{ij} + r_{ij}$$

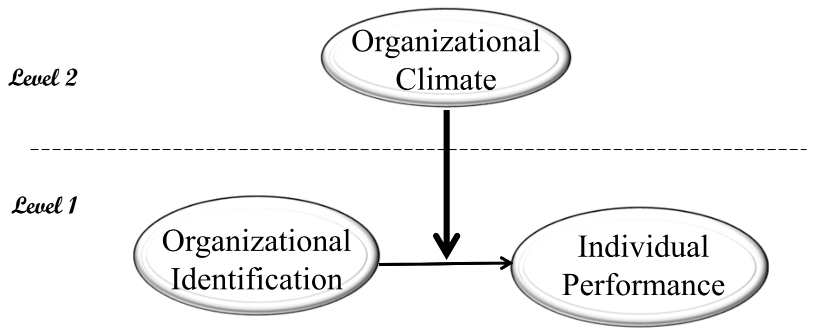
$$\beta_{0j} = \gamma_{00} + u_{0j}$$

$$\beta_{1j} = \gamma_{10} + u_{1j}$$

$$\beta_{2j} = \gamma_{20} + u_{2j}$$

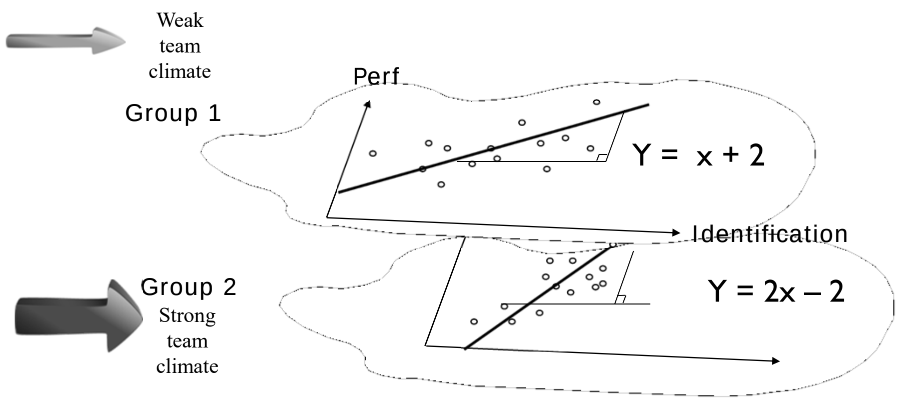
$$\beta_{3j} = \gamma_{30} + u_{3j}$$

Multi-level Moderation



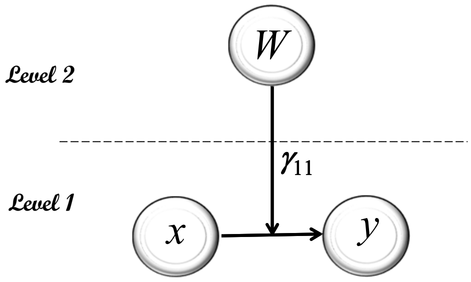
7

Hierarchical Linear Modeling



8

Multi-level Moderation



Different *intercept* for each level

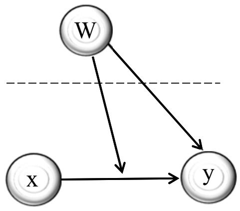
$$y_{ij} = \beta_{0j} + \beta_{1j}\left(x_{ij} - \bar{x}_{.j}\right) + r_{ij}$$
$$\beta_{0j} = \gamma_{00} + \gamma_{01} W_j + u_{0j}$$
$$\beta_{1j} = \gamma_{10} + \gamma_{11} W_j + u_{1j}$$

Different *slope* for each level

9

Bewteen-level Moderation

```
TITLE:        Sample multi-level program on helping
DATA:         FILE = helping.txt;
VARIABLE:     NAMES = group y x W;
              USEVARIABLES = y x W;
              WITHIN = x ;
            BETWEEN = W ;
              CLUSTER = group;
ANALYSIS:     TYPE = TWOLEVEL RANDOM;
MODEL:
     %WITHIN%
        s | y on x  ! call the slope of each group s
        y;   ! you can skip this line
     %BETWEEN%
        y on W;    ! Intercept predicted by W
        s on W;    ! Slope predicted by W
        y;   ! you can skip this line
OUTPUT:       TECH1;
```

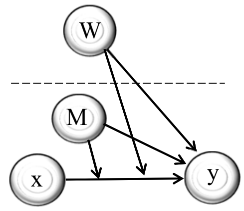


$$y_{ij} = \beta_{0j} + \beta_{1j} x_{ij} + r_{ij}$$
$$\beta_{0j} = \gamma_{00} + \gamma_{01} W_j + u_{0j}$$
$$\beta_{1j} = \gamma_{10} + \gamma_{11} W_j + u_{1j}$$

10

Between-within level Moderation

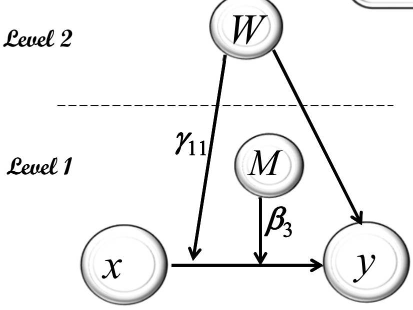
```
TITLE:      Sample multi-level program on helping
DATA:       FILE = data.txt;
VARIABLE:   NAMES = group x M xM W;
            USEVARIABLES = help;
            WITHIN = x M xM ;
            BETWEEN = W ;
            CLUSTER = group;
ANALYSIS:   TYPE = TWOLEVEL RANDOM;
MODEL:
      %WITHIN%
        s | y on x
        y on M xM;
        x M xM;
        y ! you can skip this line
      %BETWEEN%
        y on W;
        s on W;
        y;   ! you can skip this line
OUTPUT:     TECH1;
```



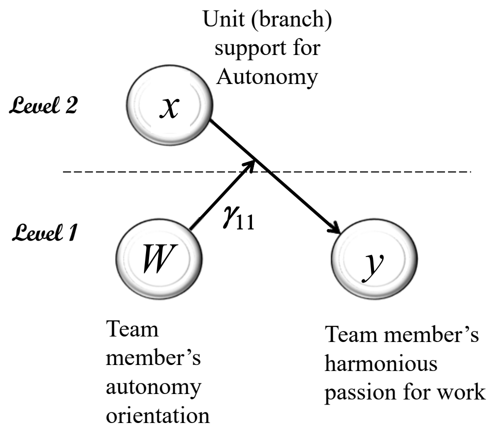
$$y_{ij} = \beta_{0j} + \beta_{1j}x_{ij} + \beta_{2j}M_{ij} + \beta_{3j}xM_{ij} + r_{ij}$$
$$\beta_{0j} = \gamma_{00} + \gamma_{01}W_j + u_{0j}$$
$$\beta_{1j} = \gamma_{10} + \gamma_{11}W +_j u_{1j}$$
$$\beta_{2j} = \gamma_{20} + u_{2j}$$
$$\beta_{3j} = \gamma_{30} + u_{3j}$$

Multi-level Moderation

$$y_{ij} = \beta_{0j} + \beta_{1j}x_{ij} + \beta_2 M_{ij} + \beta_3 M_{ij}x_{ij} + r_{ij}$$
$$\beta_{0j} = \gamma_{00} + \gamma_{01}W_j + u_{0j}$$
$$\beta_{1j} = \gamma_{10} + \gamma_{11}W_j + u_{1j}$$



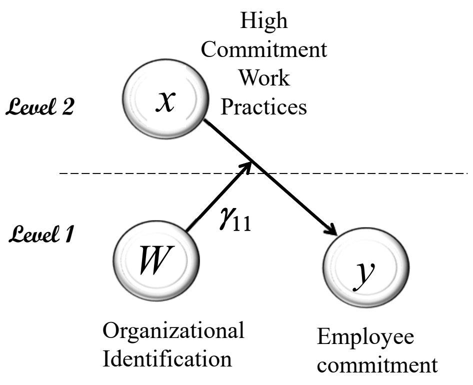
Reverse Multi-level Moderation



Source: Liu, D., Chen, X. & Yao, X. (2010) From autonomy to creativity: A multilevel investigation of the mediating role of harmonious passion. Journal of Applied Psychology.

13

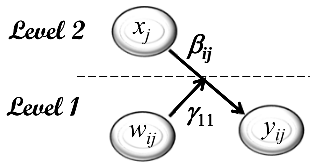
Reverse Multi-level Moderation

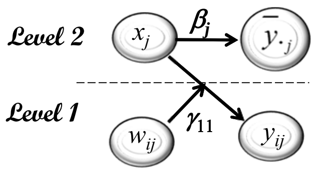


$$y_{ij} = \beta_{0j} + \beta_{1j}\left(w_{ij} - \overline{w}_{.j}\right) + r_{ij}$$
$$\beta_{0j} = \gamma_{00} + \gamma_{01}x_j + u_{0j}$$
$$\beta_{1j} = \gamma_{10} + \gamma_{11}x_j + u_{1j}$$

14

Controversy

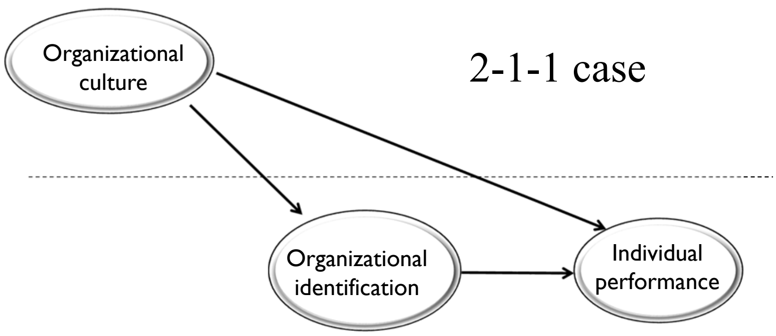




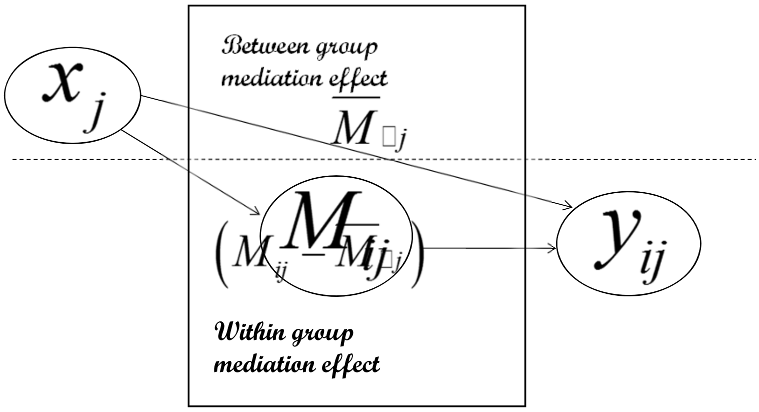
$$\beta_{ij} = \beta_j + \varepsilon_{ij}$$
$$\beta_{65} = \beta_5 + \varepsilon_{65}$$
$$\beta_{25} = \beta_5 + \varepsilon_{25}$$

15

An example of multi-level mediation



Between and within group mediation



M_{ij} = Organizational Identification of the i th employee in the j th group

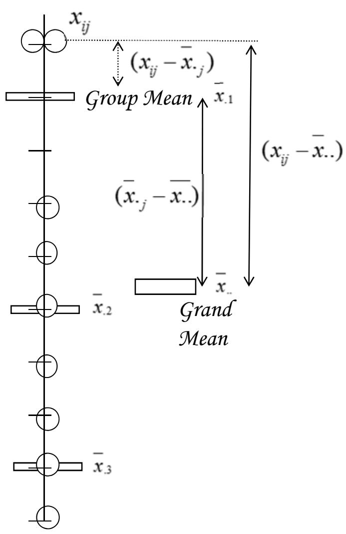
Partition of Variance and Covariance

$$\boxed{(x_{ij} - \bar{x}_{..})^2} = \boxed{(x_{ij} - \bar{x}_{.j})^2} + \boxed{(\bar{x}_{.j} - \bar{x}_{..})^2}$$

$$\text{SST} = \text{SSW} + \text{SSB}$$

$$\boxed{\frac{(x_{ij} - \bar{x}_{..})^2}{n-1}} \qquad \boxed{\frac{(x_{ij} - \bar{x}_{.j})^2}{n_j - 1}} \qquad \boxed{\frac{(\bar{x}_{.j} - \bar{x}_{..})^2}{J-1}}$$

Var Total   Var Within   Var Between



18

An example

		Grand mean	Group mean		Within score	Between score
	x_{ij}	$\bar{x}_{..}$	$\bar{x}_{.j}$	$(x_{ij}-\bar{x}_{..})$	$(x_{ij}-\bar{x}_{.j})$	$(\bar{x}_{.j}-\bar{x}_{..})$
1	3	3.11	3.00	-0.11	0.00	-0.11
1	4	3.11	3.00	0.89	1.00	-0.11
1	2	3.11	3.00	-1.11	-1.00	-0.11
2	1	3.11	1.33	-2.11	-0.33	-1.78
2	2	3.11	1.33	-1.11	0.67	-1.78
2	1	3.11	1.33	-2.11	-0.33	-1.78
3	5	3.11	5.00	1.89	0.00	1.89
3	6	3.11	5.00	2.89	1.00	1.89
3	4	3.11	5.00	0.89	-1.00	1.89

19

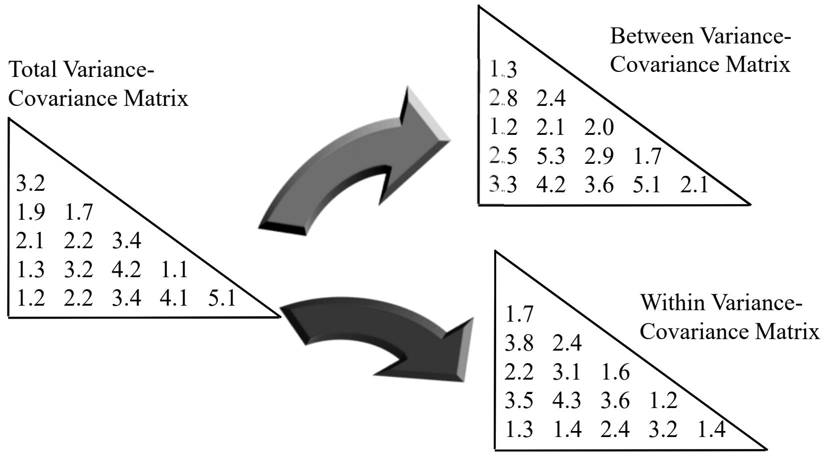
An example

	x_{ij}	$\bar{x}_{..}$	$\bar{x}_{.j}$	$(x_{ij}-\bar{x}_{..})$	$(x_{ij}-\bar{x}_{.j})$	$(\bar{x}_{.j}-\bar{x}_{..})$
1	3	3.11	3.00	-0.11	0.00	-0.11
1	4	3.11	3.00	0.89	1.00	-0.11
1	2	3.11	3.00	-1.11	-1.00	-0.11
2	1	3.11	1.33	-2.11	-0.33	-1.78
2	2	3.11	1.33	-1.11	0.67	-1.78
2	1	3.11	1.33	-2.11	-0.33	-1.78
3	5	3.11	5.00	1.89	0.00	1.89
3	6	3.11	5.00	2.89	1.00	1.89
3	4	3.11	5.00	0.89	-1.00	1.89

$\sum(x_{ij}-\bar{x}_{..})^2$	$\sum(x_{ij}-\bar{x}_{.j})^2$	$\sum(\bar{x}_{.j}-\bar{x}_{..})^2$
SST	SSW	SSB
24.89	4.67	20.22

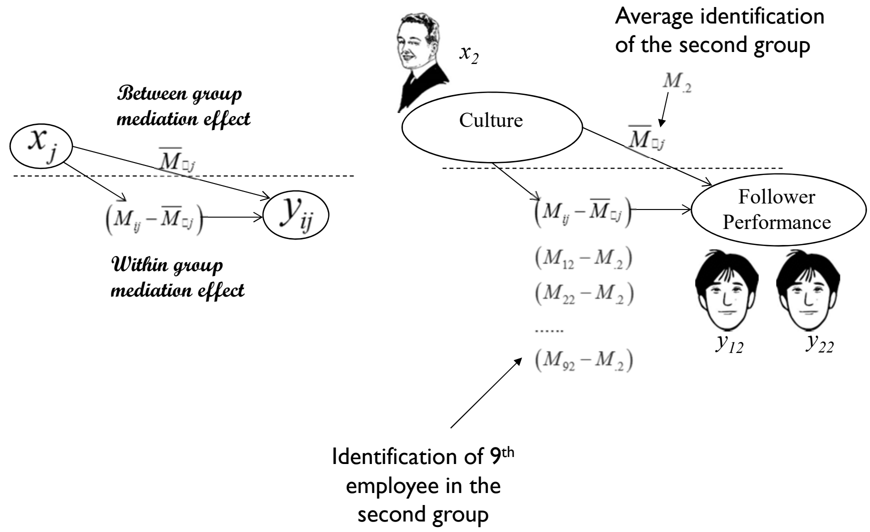
20

Partition of Variance and Covariance



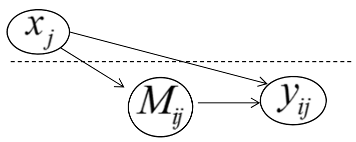
21

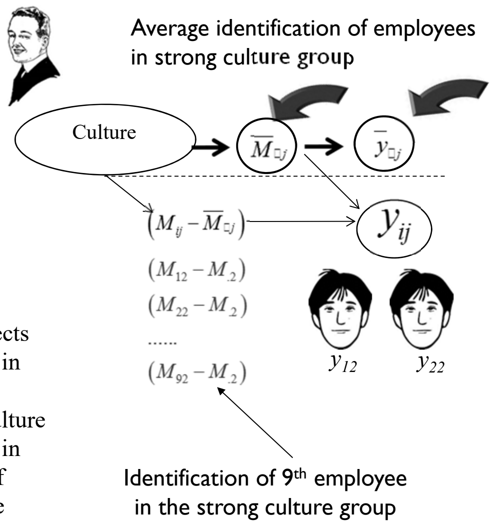
Between and within group mediation



The reality

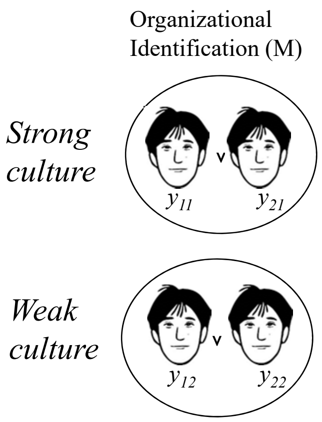
Strong culture vs. Weak culture

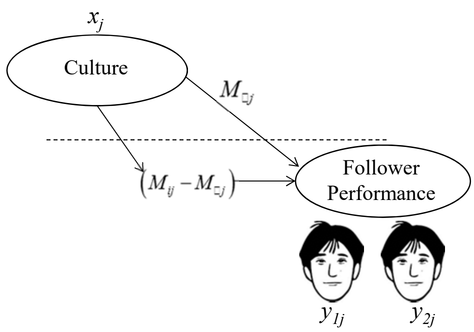




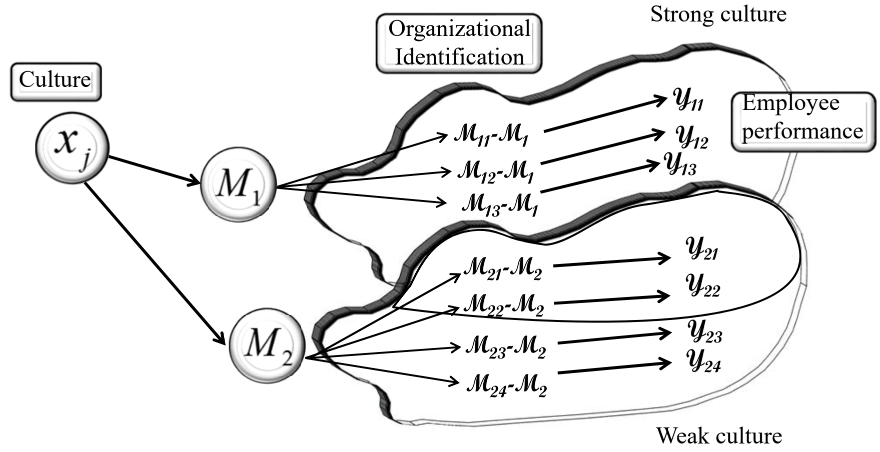
- Strong culture would have the <u>same</u> effects on identification of different employees in the strong culture group.
- Because of the same effects of strong culture on identification of different employees in the strong culture group, performance of different employees in the strong culture group are the <u>same</u> .

ANOVA 方差分析





Pseduo-Within Mediation

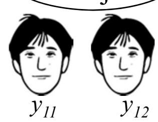


Between and within effects

x_j

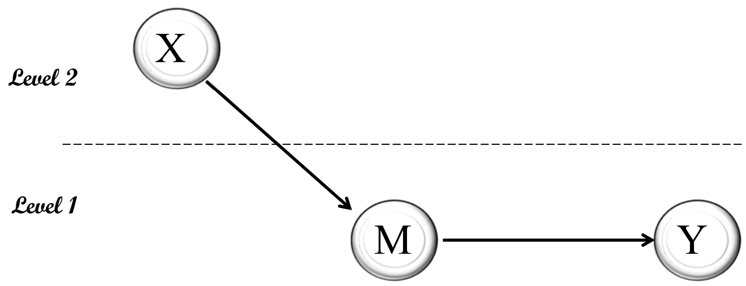
Culture $\longrightarrow \overline{M}_{\cdot j} \longrightarrow \overline{y}_{\cdot j}$

$$\left(x_{ij} - x_{\cdot j}\right) \longrightarrow \left(M_{ij} - \overline{M}_{\cdot j}\right) \longrightarrow y_{ij}$$

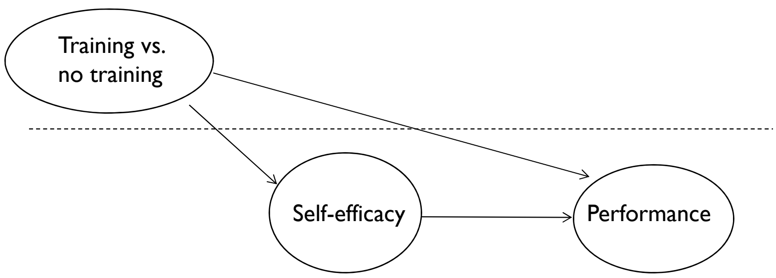


$$y_{ij} = \gamma_{00} + \gamma_{01} x_j + \gamma_{02} \overline{M}_{\cdot j} + \gamma_{03} \left(M_{ij} - \overline{M}_{\cdot j}\right) + \varepsilon_{ij}$$

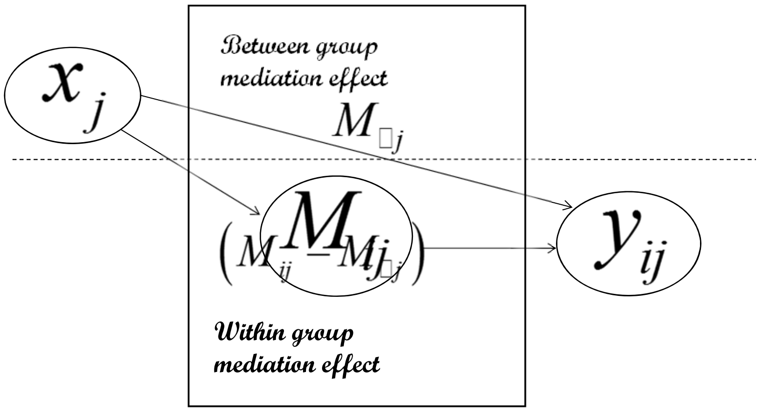
2-1-1 case



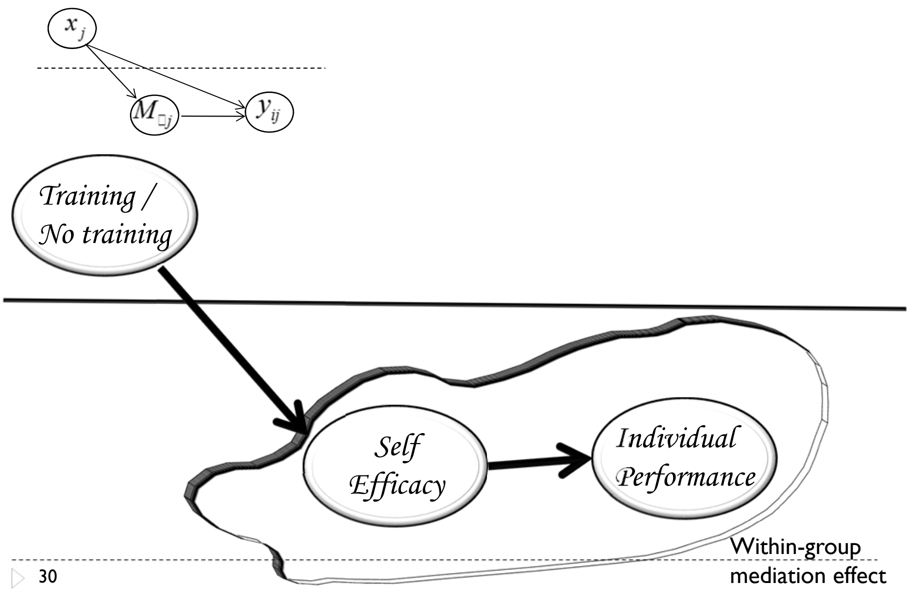
An example of multi-level mediation



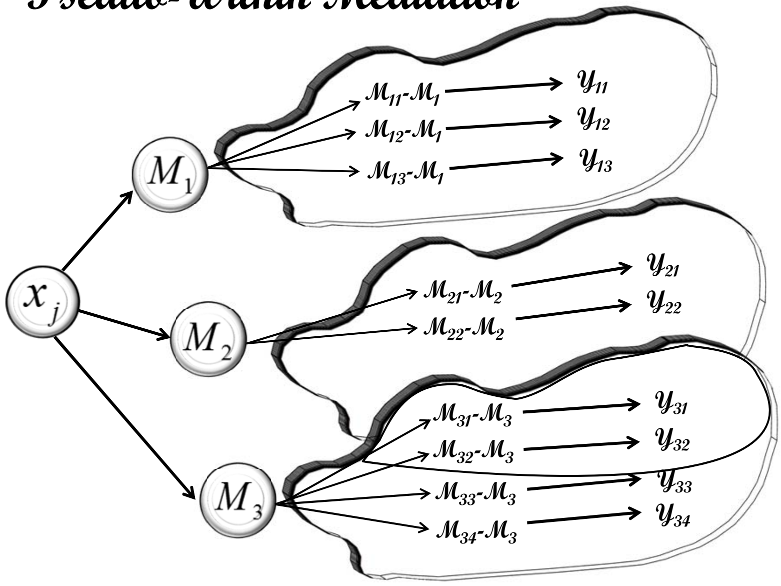
Between and within group mediation



Traditional MLM

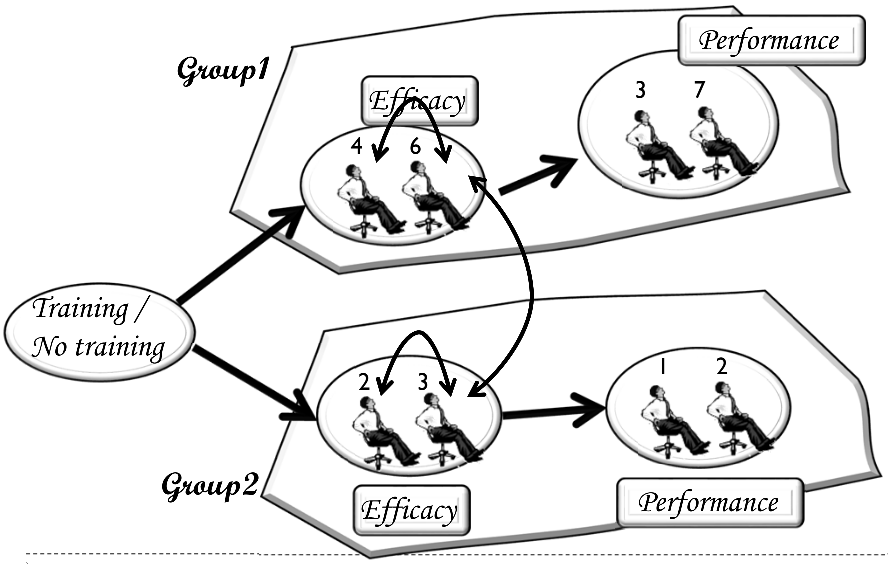


Pseduo-Within Mediation



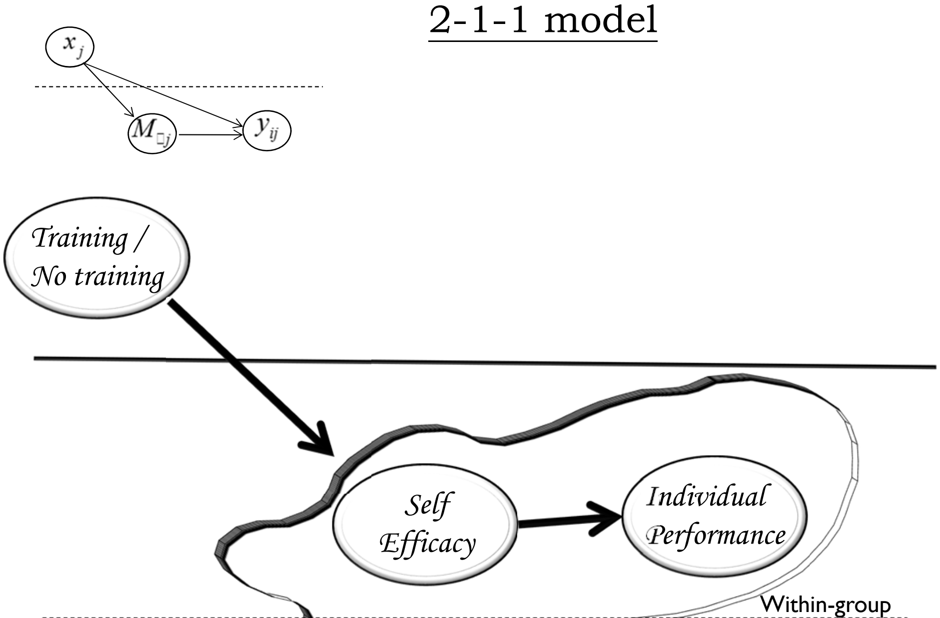
31

Within and Between mediation



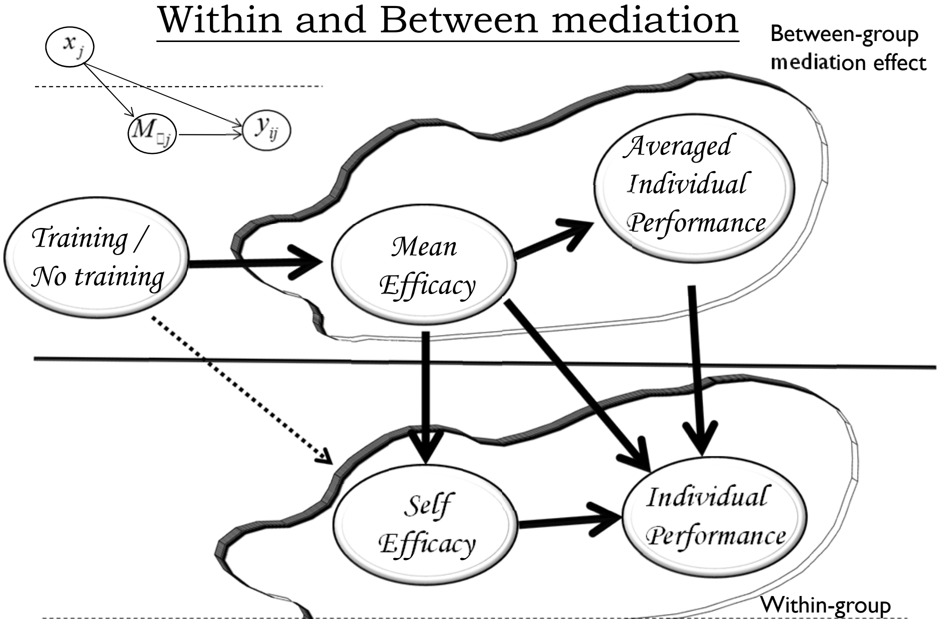
32

2-1-1 model



33

Within and Between mediation

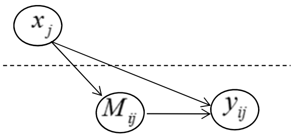


34

B&K procedure in a 2-1-1 case

Step 1:

$$y_{ij} = \beta_{0j}^{(1)} + r_{ij} \quad (1)$$
$$\beta_{0j}^{(1)} = \gamma_{00}^{(1)} + \gamma_{01}^{(1)} x_j + u_{0j}^{(1)} \quad (2)$$



Step 2:

$$y_{ij} = \beta_{0j}^{(3)} + \beta_{1j}^{(3)} M_{ij} + r_{ij}^{(3)} \quad (5)$$
$$\beta_{0j}^{(3)} = \gamma_{00}^{(3)} + \gamma_{01}^{(3)} x_j + u_{0j}^{(3)} \quad (6)$$
$$\beta_{1j}^{(3)} = \gamma_{10}^{(3)} \quad (7)$$

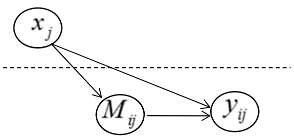
$$H_o = \gamma_{01}^{(1)} - \gamma_{01}^{(3)}$$

Not suggested

B&K procedure in a 2-1-1 case

Step 1:

$$y_{ij} = \beta_{0j}^{(1)} + r_{ij} \quad (1)$$
$$\beta_{0j}^{(1)} = \gamma_{00}^{(1)} + \gamma_{01}^{(1)} x_j + u_{0j}^{(1)} \quad (2)$$



Step 2:

$$y_{ij} = \beta_{0j}^{(4)} + \beta_{1j}^{(4)} (M_{ij} - M_{\cdot j}) + r_{ij}^{(4)} \quad (8)$$
$$\beta_{0j}^{(3)} = \gamma_{00}^{(4)} + \gamma_{01}^{(4)} x_j + \gamma_{02}^{(4)} M_{\cdot j} + u_{0j}^{(4)} \quad (9)$$
$$\beta_{1j}^{(4)} = \gamma_{10}^{(4)} \quad (10)$$

$$H_o = \gamma_{01}^{(1)} - \gamma_{01}^{(4)}$$

B&K Testing mediation in a 2-1-1 case

Step 1: X→Y

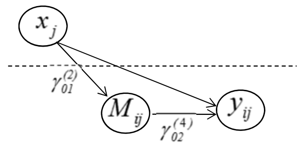
$$y_{ij} = \beta_{0j}^{(1)} + r_{ij} \qquad (1)$$
$$\beta_{0j}^{(1)} = \gamma_{00}^{(1)} + \gamma_{01}^{(1)} x_j + u_{0j}^{(1)} \qquad (2)$$

Step 2: X→M

$$M_{ij} = \beta_{0j}^{(2)} + r_{ij}^{(2)} \qquad (3)$$
$$\beta_{0j}^{(2)} = \gamma_{00}^{(2)} + \gamma_{01}^{(2)} x_j + u_{0j}^{(2)} \qquad (4)$$

Step 3: X→M→Y

$$y_{ij} = \beta_{0j}^{(4)} + \beta_{1j}^{(4)}\left(M_{ij} - M_{\cdot j}\right) + r_{ij}^{(4)} \qquad (8)$$
$$\beta_{0j}^{(3)} = \gamma_{00}^{(4)} + \gamma_{01}^{(4)} x_j + \gamma_{02}^{(4)} M_{\cdot j} + u_{0j}^{(4)} \qquad (9)$$
$$\beta_{1j}^{(4)} = \gamma_{10}^{(4)} \qquad (10)$$



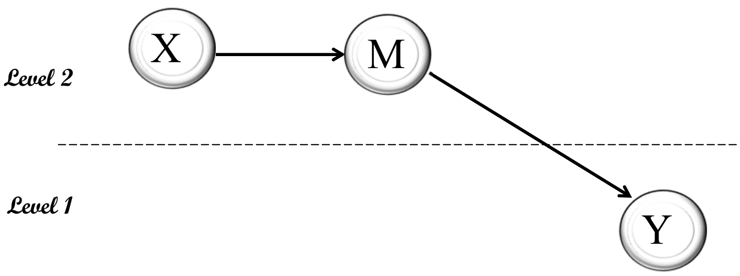
Between mediation

Within mediation is not too meaningful in this case because all individual variations within group with a group level x_j should be considered as random.

$$H_1 = \gamma_{01}^{(1)} - \gamma_{02}^{(4)}$$

$$H_2 = \gamma_{01}^{(2)} * \gamma_{02}^{(4)}$$

2-2-1 case



38

Grand Mean Centering in a 2-2-1 case

Step 1: X→Y

$$y_{ij} = \beta_{0j}^{(1)} + r_{ij} \qquad (11)$$

$$\beta_{0j}^{(1)} = \gamma_{00}^{(1)} + \gamma_{01}^{(1)} x_j + u_{0j}^{(1)} \qquad (12)$$

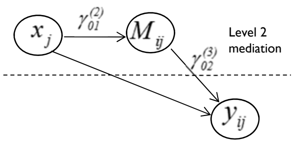
Step 2: X→M

$$M_j = \gamma_{00}^{(2)} + \gamma_{01}^{(2)} x_j + u_{0j}^{(2)} \qquad (14)$$

Step 3: X→M→Y

$$y_{ij} = \beta_{0j}^{(3)} + r_{ij}^{(3)} \qquad (15)$$

$$\beta_{0j}^{(3)} = \gamma_{00}^{(3)} + \gamma_{01}^{(3)} x_j + \gamma_{02}^{(3)} M_j + u_{0j}^{(3)} \qquad (16)$$

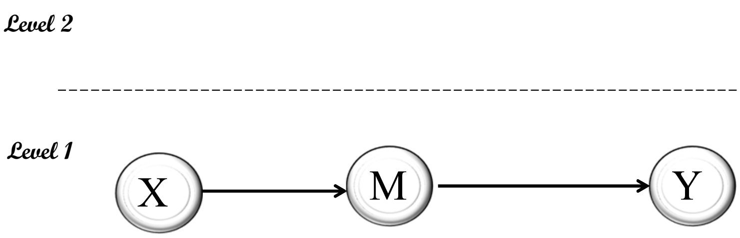


$$H_1 = \gamma_{01}^{(1)} - \gamma_{01}^{(3)}$$

$$H_2 = \gamma_{01}^{(2)} * \gamma_{02}^{(3)}$$

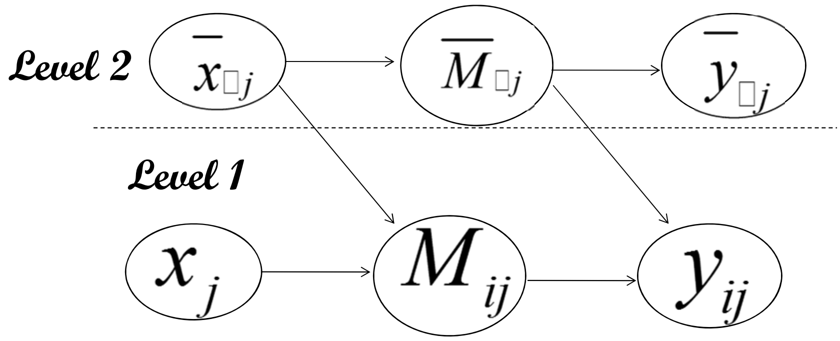
Grand mean centering and Group mean centering are equally valid in this case. (No confounding of mediation effect)

1-1-1 case



40

1-1-1Mediation in multi-level context



Testing mediation in a 1-1-1 case (Multi-level context)

$$y_{ij} = \beta_{0j}^{(4)} + \beta_{1j}^{(4)}\left(x_{ij} - x_{\cdot j}\right) + r_{ij}^{(4)}$$
$$\beta_{0j}^{(4)} = \gamma_{00}^{(4)} + \gamma_{01}^{(4)} x_{\cdot j} + u_{0j}^{(4)}$$
$$\beta_{1j}^{(4)} = \gamma_{10}^{(4)}$$

$$x_j \rightarrow M_{ij} \rightarrow y_{ij}$$

Level 1 mediation

$$M_{ij} = \beta_{0j}^{(5)} + \beta_{1j}^{(5)}\left(x_{ij} - x_{\cdot j}\right) + r_{ij}^{(5)}$$
$$\beta_{0j}^{(5)} = \gamma_{00}^{(5)} + \gamma_{01}^{(5)} x_{\cdot j} + u_{0j}^{(5)}$$
$$\beta_{1j}^{(5)} = \gamma_{10}^{(5)}$$

$$H_0 = \gamma_{10}^{(4)} - \gamma_{10}^{(6)}$$
$$H_0 = \gamma_{10}^{(5)} * \gamma_{20}^{(6)}$$

Level 2 mediation

$$y_{ij} = \beta_{0j}^{(6)} + \beta_{1j}^{(6)}\left(x_{ij} - x_{\cdot j}\right) + \beta_{2j}^{(6)}\left(M_{ij} - M_{\cdot j}\right) + r_{ij}^{(6)}$$
$$\beta_{0j}^{(6)} = \gamma_{00}^{(6)} + \gamma_{01}^{(6)} x_{\cdot j} + \gamma_{02}^{(6)} M_{\cdot j} + u_{0j}^{(6)}$$
$$\beta_{1j}^{(6)} = \gamma_{10}^{(6)}$$
$$\beta_{2j}^{(6)} = \gamma_{20}^{(6)}$$

$$H_0 = \gamma_{01}^{(4)} - \gamma_{01}^{(6)}$$
$$H_0 = \gamma_{01}^{(5)} * \gamma_{02}^{(6)}$$

Mplus syntax: Simple mediation

```
TITLE: simple mediation
DATA: FILE IS mydata.dat; ! text file containing raw data in long format
VARIABLE: NAMES ARE x m y;
USEVARIABLES ARE x m y;
ANALYSIS: BOOTSTRAP IS 5000; ! bootstrap is recommended for simple mediation
MODEL: ! model specification follows
    m ON x; ! regress mediator on independent variable
    y ON x m; ! regress outcome on both mediator and independent variable
MODEL INDIRECT: ! request significance test for indirect effect of x on y via m
    y IND m x; ! indirect effect of interest (ending in y and starting with x)
OUTPUT: CINTERVAL(BCBOOTSTRAP);
! request bias-corrected bootstrap
! confidence intervals
```

$$m_i = \beta_1 + a\, x_i + \varepsilon_i$$
$$y_i = \beta_2 + b\, m_i + r_i$$

Source: http://www.quantpsy.org/pubs/syntax_appendix_081311.pdf

43

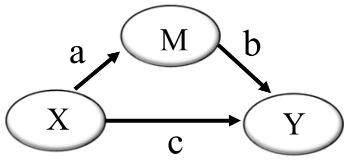
Bootstrapping

- bootstrap is a strap that is looped and sewn to the top of a boot for pulling it on.
- bootstrapping usually refers to a self-starting process that is supposed to proceed without external input.

Statistics	Sampling distribution	Statistical inference
mean	Normal distribution	Z-test
MSR	F-distribution	F-test
β coefficient	T-distribution	t-test
$a*b$ (mediation)	???	???

44

Testing mediator using SEM



Sobel test

http://people.ku.edu/~preacher/sobel/sobel.htm

(*when N is large*) $H_o : ab = 0$

$H_o : c = 0$

$$S_{\hat{a}\hat{b}} = \sqrt{\hat{a}^2 s_{\hat{b}}^2 + \hat{b}^2 s_{\hat{a}}^2 + s_{\hat{a}}^2 s_{\hat{b}}^2}$$

$$\hat{c} \pm S_{\hat{c}} \, t_{N-2}$$

$$\hat{a}\hat{b} \pm S_{\hat{a}\hat{b}} \, z_{\hat{a}\hat{b}}$$

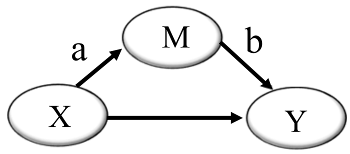
Source: Shrout, P.E., & Bolger, N. (2002) Mediation in experimental and nonexperimental studies: New procedures and recommendations. Psychological Methods, 7(4), 422-445.

45

46

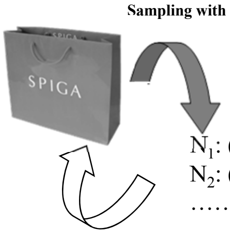
Bootstrapping: Develop sub-samples

$$H_0 : ab = 0$$

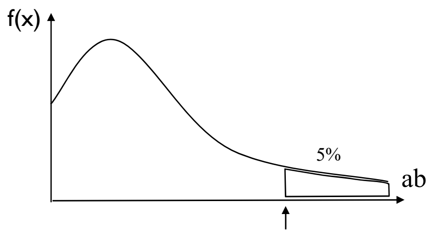


N=200
Your sample

N=200
Sampling with replacement



N_1: $(ab)_1$
N_2: $(ab)_2$
……
……
……
N_{10000}: $(ab)_{10000}$



Critical value of sample ab to reject H_o:ab=0

47

Bootstrapping

$x_1 \xrightarrow{a} x_2 \xrightarrow{b} x_3$

Sample

	x_1	x_2	x_3
1	3	4	6
2	3	2	5
3	4	4	3
4	5	6	4
5	1	2	1
6	2	3	1
7	4	3	2
8	1	3	4
9	3	3	2
10	1	1	2

$r_{12} = .76 \quad r_{23} = .38$

$ab = .29$

Sample one

	x_1	x_2	x_3
1	3	4	6
2	3	4	6
3	3	4	6
4	1	1	2
5	2	3	1
6	4	4	3
7	4	3	2
8	5	6	4
9	4	4	3
10	3	2	5

$r_{12} = .80 \quad r_{23} = .35$

$ab = .28$

Sample two

	x_1	x_2	x_3
1	1	2	1
2	2	3	1
3	2	3	1
4	3	3	2
5	5	6	4
6	4	4	3
7	3	4	6
8	4	3	2
9	4	4	3
10	3	3	2

$r_{12} = .82 \quad r_{23} = .68$

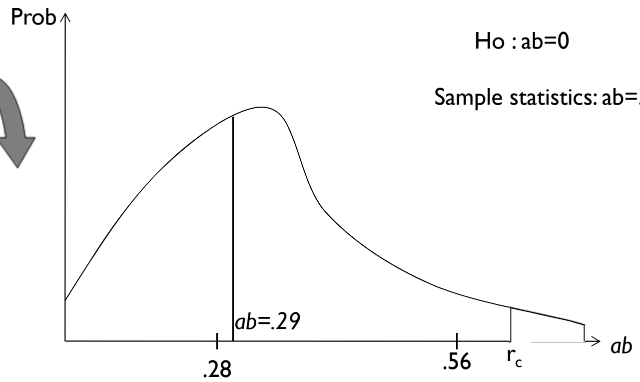
$ab = .56$

。 。 。

48

Bootstrapping

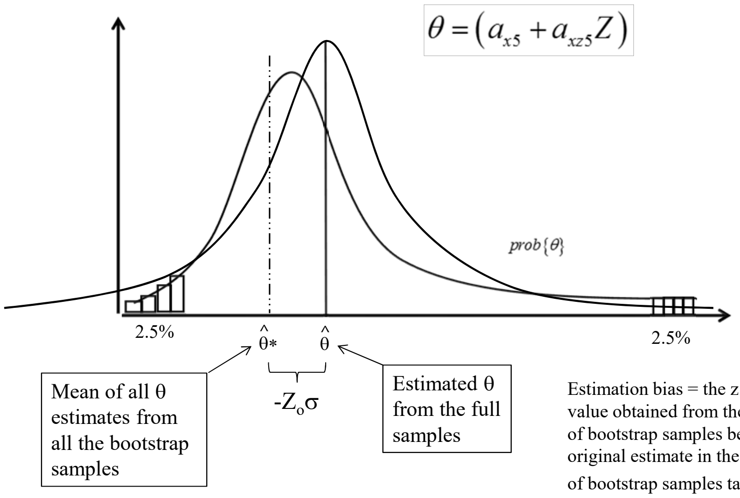
$x_1 \xrightarrow{a} x_2 \xrightarrow{b} x_3$



Ho : ab=0

Sample statistics: ab=.29

Biased corrected bootstrapping estimates



Estimation bias = the z score of the value obtained from the proportion of bootstrap samples below the original estimate in the total number of bootstrap samples taken.

49

Bias corrected bootstrapping estimates

The basic estimates

The basic bootstrap confidence limits were obtained with the percentile method as described by Efron and Tibshirani (1993). The sample parameter values at the $\alpha/2$ and $(1-\alpha/2)$ percentile of the bootstrap sampling distribution were used as the lower and upper confidence limits. For example, the percentile method 90% confidence limits would be the values of the bootstrap sampling distribution at 5% and 95% cumulative frequency. (p.114)

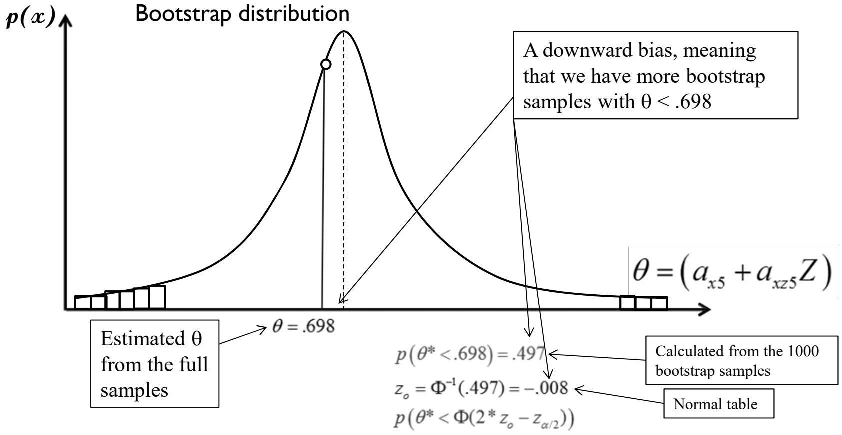
The bias-corrected estimates

This method corrects for bias in the central tendency of the estimates. This bias is expressed by $\hat{z}_o$, which is the z score of the value obtained from the proportion of bootstrap samples below the original estimate in the total number of bootstrap samples taken. In other words, $\hat{z}_o$ is the z score of the percentile of the observed sample indirect effect. The upper confidence limit was then found as the z score of $2\hat{z}_o+z_{(1-\alpha/2)}$ and the lower limit was $2\hat{z}_o+z_{\alpha/2}$. (p.115)

MacKinnon, D. P., Lockwood, C. M., & Williams, J. (2004). Confidence limits for the indirect effect: Distribution of the product and resampling methods. Multivariate Behavioral Research, 39, 99–128.

50

Bias corrected bootstrapping estimates



$p(\theta^* < .698) = .497$

$z_o = \Phi^{-1}(.497) = -.008$

$p(\theta^* < \Phi(2 * z_o - z_{\alpha/2}))$

$= p(\theta^* < \Phi(2 * z_o - 1.96))$

$= p(\theta^* < \Phi(-1.976))$

$= .0241$

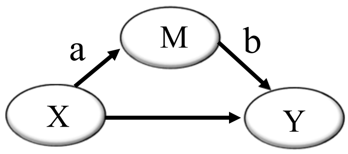
$\theta = .636$

Stine, R. (1989) An introduction to bootstrap methods: Examples and ideas. Sociological Methods and Research, 18, No. 2&3, 243-291

51

Testing mediator using Mplus

```
TITLE: Example of Mediation using Mplus
   DATA:
   FILE IS OS-902.dat;
   FORMAT is FREE;
   VARIABLE:
   names are x m y;
   usevariables are x m y;
   ANALYSIS:
   bootstrap = 1000;
   MODEL:
   m on x (a);  <----------  x→m (path a)
   y on m (b)   <----------  m→y (path b)  Note: no ; after this statement
   x;
  MODEL CONSTRAINT:
  new (ind);
  ind = a*b;    <----------  Define a new variable called ind = ab
  output:
  cinterval (bcbootstrap);
```



52

Mplus Output

C:\Windows\system32\cmd.exe

```
ESTIMATION WITH BOOTSTRAP DRAW NUMBER 190
ESTIMATION WITH BOOTSTRAP DRAW NUMBER 191
ESTIMATION WITH BOOTSTRAP DRAW NUMBER 192
ESTIMATION WITH BOOTSTRAP DRAW NUMBER 193
ESTIMATION WITH BOOTSTRAP DRAW NUMBER 194
ESTIMATION WITH BOOTSTRAP DRAW NUMBER 195
ESTIMATION WITH BOOTSTRAP DRAW NUMBER 196
ESTIMATION WITH BOOTSTRAP DRAW NUMBER 197
ESTIMATION WITH BOOTSTRAP DRAW NUMBER 198
ESTIMATION WITH BOOTSTRAP DRAW NUMBER 199
ESTIMATION WITH BOOTSTRAP DRAW NUMBER 200
ESTIMATION WITH BOOTSTRAP DRAW NUMBER 201
ESTIMATION WITH BOOTSTRAP DRAW NUMBER 202
ESTIMATION WITH BOOTSTRAP DRAW NUMBER 203
ESTIMATION WITH BOOTSTRAP DRAW NUMBER 204
ESTIMATION WITH BOOTSTRAP DRAW NUMBER 205
ESTIMATION WITH BOOTSTRAP DRAW NUMBER 206
ESTIMATION WITH BOOTSTRAP DRAW NUMBER 207
ESTIMATION WITH BOOTSTRAP DRAW NUMBER 208
ESTIMATION WITH BOOTSTRAP DRAW NUMBER 209
ESTIMATION WITH BOOTSTRAP DRAW NUMBER 210
ESTIMATION WITH BOOTSTRAP DRAW NUMBER 211
ESTIMATION WITH BOOTSTRAP DRAW NUMBER 212
ESTIMATION WITH BOOTSTRAP DRAW NUMBER 213
ESTIMATION WITH BOOTSTRAP DRAW NUMBER 214
```


Mplus Output

```
Mplus VERSION 7.11
MUTHEN & MUTHEN
05/22/2014   9:21 AM

INPUT INSTRUCTIONS
 TITLE: Example of Mediation using Mpls
      DATA:
      FILE IS OS-902.dat;
      FORMAT is FREE;
      VARIABLE:
      names are x m y;
      usevariables are x m y;
      ANALYSIS:
      bootstrap = 1000;
      MODEL:
      m on x (a);
      y on m (b);
      x;
      MODEL CONSTRAINT:
      new (ind);
      ind = a*b;
      output:
      cinterval (bcbootstrap);


INPUT READING TERMINATED NORMALLY
```

```
SUMMARY OF ANALYSIS

Number of groups                                     1
Number of observations                             962
Number of dependent variables                        2
Number of independent variables                      1
Number of continuous latent variables                0

Observed dependent variables
  Continuous
   M      Y
Observed independent variables
   X

Estimator                                           ML
Information matrix                            OBSERVED
Maximum number of iterations                      1000
Convergence criterion                        0.500D-04
Maximum number of steepest descent iterations       20
Number of bootstrap draws
    Requested                                     1000
    Completed                                     1000
Input data file(s)
  OS-902.dat

Input data format  FREE

THE MODEL ESTIMATION TERMINATED NORMALLY
```

55

Mplus Output

MODEL FIT INFORMATION

Number of Free Parameters		7
Loglikelihood		
H0 Value	-2438.903	
H1 Value	-2438.903	
Information Criteria		
Akaike (AIC)	4891.805	
Bayesian (BIC)	4925.888	
Sample-Size Adjusted BIC (n* = (n + 2) / 24)	4903.656	
Chi-Square Test of Model Fit		
Value	0.000	
Degrees of Freedom		0
P-Value	0.0000	
RMSEA (Root Mean Square Error Of Approximation)		
Estimate	0.000	
90 Percent C.I.	0.000 0.000	
Probability RMSEA <= .05	0.000	
CFI/TLI		
CFI	1.000	
TLI	1.000	
Chi-Square Test of Model Fit for the Baseline Model		
Value	121.454	
Degrees of Freedom		3
P-Value	0.0000	
SRMR (Standardized Root Mean Square Residual)		
Value	0.000	

Model χ^2 and d.f. → Chi-Square Test of Model Fit

RMSEA → RMSEA

CFI , TLI → CFI/TLI

SRMR → SRMR

56

Mplus Output

MODEL RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
M ON				
X	-0.397	0.042	-9.420	0.000
Y ON				
M	-0.024	0.027	-0.880	0.379
X	0.153	0.039	3.959	0.000
Intercepts				
M	5.449	0.132	41.139	0.000
Y	3.135	0.196	16.017	0.000
Residual Variances				
M	0.899	0.042	21.413	0.000
Y	0.607	0.021	28.805	0.000
New/Additional Parameters				
IND	0.010	0.011	0.867	0.386

x→m (parameter *a*) → M ON X

m→y (parameter *b*) → Y ON M

IND is *ab* → IND

Sobel Test

57

Mplus Output

CONFIDENCE INTERVALS OF MODEL RESULTS

MODEL CONSTRAINT:
new (ind);
ind = a*b;
output:
cinterval (bcbootstrap);

CONFIDENCE INTERVALS OF MODEL RESULTS

	Lower .5%	Lower 2.5%	Lower 5%	Estimate	Upper 5%	Upper 2.5%	Upper .5%
M ON							
X	-0.493	-0.471	-0.458	-0.397	-0.318	-0.302	-0.278
Y ON							
M	-0.122	-0.110	-0.102	-0.061	-0.017	-0.010	0.010
Means							
X	3.001	3.016	3.023	3.063	3.103	3.109	3.125
Intercepts							
M	5.052	5.142	5.195	5.449	5.635	5.676	5.743
Y	3.449	3.537	3.571	3.758	3.927	3.950	3.999
Variances							
X	0.536	0.551	0.558	0.599	0.640	0.648	0.660
Residual Variances							
M	0.800	0.817	0.833	0.899	0.967	0.981	1.015
Y	0.567	0.581	0.587	0.620	0.660	0.666	0.680
New/Additional Parameters							
IND	-0.006	0.003	0.007	0.024	0.043	0.046	0.055

New/Additional Parameters

58

Mplus Output

CONFIDENCE INTERVALS OF MODEL RESULTS

Bootstrapping estimated confidence interval of parameter *ab*

	Lower .5%	Lower 2.5%	Lower 5%	Estimate	Upper 5%	Upper 2.5%	Upper .5%
M ON							
X	-0.493	-0.471	-0.458	-0.397	-0.318	-0.302	-0.278
Y ON							
M	-0.092	-0.075	-0.067	-0.024	0.020	0.031	0.045
X	0.052	0.080	0.092	0.153	0.222	0.232	0.250
Intercepts							
M	5.052	5.142	5.195	5.449	5.635	5.676	5.743
Y	2.579	2.716	2.794	3.135	3.437	3.490	3.578
Residual Variances							
M	0.800	0.817	0.833	0.899	0.967	0.981	1.015
Y	0.553	0.571	0.577	0.607	0.647	0.657	0.665
New/Additional Parameters							
IND	-0.018	-0.012	-0.007	0.010	0.028	0.031	0.037

2-1-1 model (single level analysis)

```
TITLE: simple mediation
DATA: FILE IS mydata.dat; ! text file containing raw data in long format
VARIABLE: NAMES ARE x m y;
USEVARIABLES ARE x m y;
ANALYSIS: BOOTSTRAP IS 5000; ! bootstrap is recommended for simple mediation
MODEL: ! model specification follows
    m ON x; ! regress mediator on independent variable
    y ON x m; ! regress outcome on both mediator and independent variable
MODEL INDIRECT: ! request significance test for indirect effect of x on y via m
    y IND m x; ! indirect effect of interest (ending in y and starting with x)
OUTPUT: CINTERVAL(BCBOOTSTRAP);
! request bias-corrected bootstrap
! confidence intervals
```

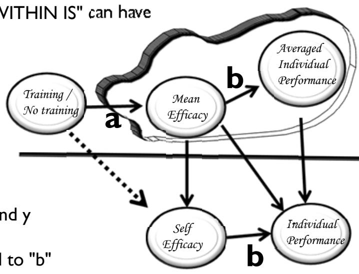
$$m_i = \beta_1 + a\,x_i + \varepsilon_i$$
$$y_i = \beta_2 + b\,m_i + r_i$$

Source: http://www.quantpsy.org/pubs/syntax_appendix_081311.pdf

2-1-1 model (traditional MLM)

```
TITLE: 2-1-1 mediation (traditional MLM)
DATA: FILE IS mydata.dat; ! text file containing raw data in long format
VARIABLE: NAMES ARE group x m y;
USEVARIABLES ARE group x m y;
BETWEEN IS x; ! identify variables with only Between variance;
    !variables that are not claimed as "BETWEEN IS" or "WITHIN IS" can have
    !both Within and Between variance
CLUSTER IS group; ! Level-2 grouping identifier
ANALYSIS: TYPE IS TWOLEVEL RANDOM;
MODEL: ! model specification follows
%WITHIN% ! Model for Within effects follows
    m y; ! estimate Level-1 (residual) variances for m and y
    y ON m (b); ! regress y on m, call the slope "b"
%BETWEEN% ! Model for Between effects follows
    x m y; ! estimate Level-2 (residual) variances for x, m, and y
    m ON x (a); ! regress m on x, call the slope "a"
    y ON m (b); ! regress y on m, constrain the slope equal to "b"
    y ON x; ! regress y on x
MODEL CONSTRAINT: ! section for computing indirect effect
    NEW(indb); ! name the indirect effect
        indb=a*b; ! compute the Between indirect effect
OUTPUT:TECH1 TECH8 CINTERVAL; ! request parameter specifications, starting values,
! optimization history, and confidence intervals for all effects
```

Equalize between and within mediation

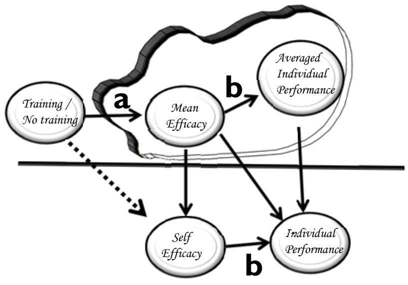


 Source: http://www.quantpsy.org/pubs/syntax_appendix_081311.pdf

2-1-1 model (traditional MLM)

```
MODEL:
%WITHIN%
    m y;
    y ON m (b);

%BETWEEN%
    x m y;
    m ON x (a);
    y ON m (b);
    y ON x;

MODEL CONSTRAINT:
NEW(indb);
    indb=a*b;
```

Equalize between and within mediation



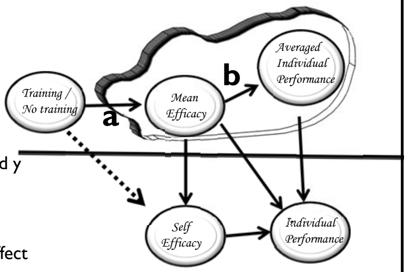
$$y_{ij} = \beta_{0j} + \beta_{1j} m_{ij} + r_{ij}$$
$$\beta_{0j} = \gamma_{00} + u_{0j}$$
$$\beta_{1j} = b$$
$$m_{\cdot j} = \alpha_0 + a x_j + \varepsilon_j$$

Source: http://www.quantpsy.org/pubs/syntax_appendix_081311.pdf

2-1-1 model (MSEM)

```
TITLE: 2-1-1 mediation (MSEM)
DATA: FILE IS mydata.dat; ! text file containing raw data in long format
VARIABLE: NAMES ARE group x m y;
USEVARIABLES ARE group x m y;
BETWEEN IS x; ! identify variables with only Between variance;  variables that are not claimed as
              "BETWEEN IS" or "WITHIN IS" can have both Within and Between variance
CLUSTER IS group; ! Level-2 grouping identifier
ANALYSIS:TYPE IS TWOLEVEL RANDOM;
MODEL: ! model specification follows
%WITHIN% ! Model for Within effects follows
    m y; ! estimate Level-1 (residual) variances for m and y
    y ON m; ! regress y on m
%BETWEEN% ! Model for Between effects follows
    x m y; ! estimate Level-2 (residual) variances for x, m, and y
    m ON x(a); ! regress m on x, call the slope "a"
    y ON m (b); ! regress y on m, call the slope "b"
    y ON x; ! regress y on x
MODEL CONSTRAINT: ! section for computing indirect effect
    NEW(indb); ! name the indirect effect
    indb=a*b; ! compute the Between indirect effect
OUTPUT:TECH1 TECH8 CINTERVAL; ! request parameter specifications, starting values,
                              ! optimization history, and confidence intervals for all effects
```

Separate between and within mediation



Source: http://www.quantpsy.org/pubs/syntax_appendix_081311.pdf

2-1-1 model (MSEM)

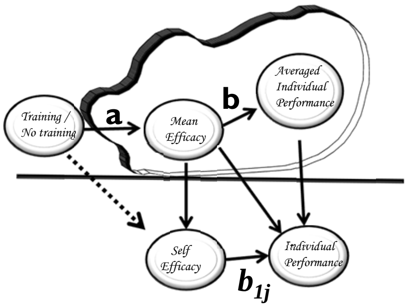
```
MODEL:
%WITHIN%
   m y;
   y ON m;
```

Free estimate of b_{1j} on each group

```
%BETWEEN%
   x m y;
   m ON x (a);
   y ON m (b);
   y ON x;
```

Separate between and within mediation

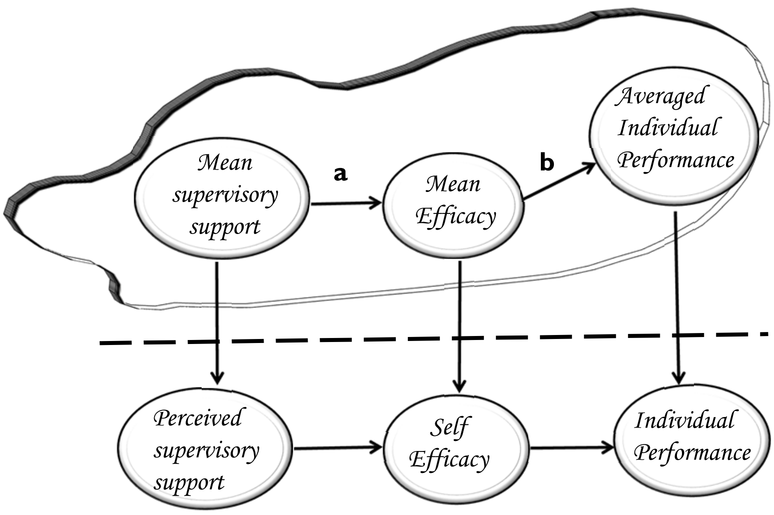
```
MODEL CONSTRAINT:
   NEW(indb);
   indb=a*b;
```



$$y_{ij} = b_{0j} + b_{1j} m_{ij} + r_{ij}$$
$$b_{0j} = \gamma_{00} + u_{0j}$$
$$b_{1j} = \gamma_{10} + u_{1j}$$
$$m_{\cdot j} = \alpha_0 + a x_{\cdot j} + \varepsilon_j$$
$$y_{\cdot j} = \alpha_0 + b m_{\cdot j} + \varepsilon_j$$

Source: http://www.quantpsy.org/pubs/syntax_appendix_081311.pdf

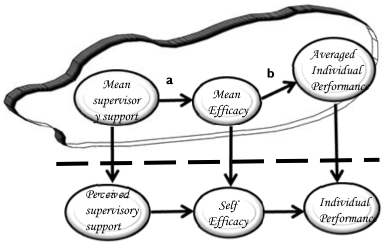
1-1-1 model (MSEM)



1-1-1 model (MSEM, fixed slope)

```
TITLE: 1-1-1 mediation (MSEM)
DATA: FILE IS mydata.dat; ! text file containing raw data in long format
VARIABLE: NAMES ARE group x m y;
USEVARIABLES ARE group x m y;
BETWEEN IS ; ! No variable with only Between variance;  variables that are not claimed as
        "BETWEEN IS" or "WITHIN IS" can have both Within and Between variance
CLUSTER IS group;
ANALYSIS: TYPE IS TWOLEVEL RANDOM;
MODEL:
%WITHIN%
    x m y;
    m ON x (aw); ! Fixed slope for all groups
    y ON m (bw); ! Fixed slope for all groups
    y ON x;  ! direct effect
%BETWEEN%
    x m y;
    m ON x (ab);
    y ON m (bb);
    y ON x;
MODEL CONSTRAINT:
    NEW(indb indw);
    indw=aw*bw;
    indb=ab*bb;
 OUTPUT: TECH1 TECH8 CINTERVAL;
```

Separate between and within mediation



65 Source: http://www.quantpsy.org/pubs/syntax_appendix_081311.pdf

1-1-1 model (MSEM, fixed slope)

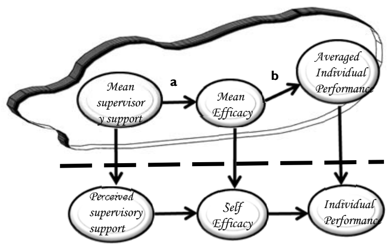
```
MODEL:
%WITHIN%
    x m y;
    m ON x (aw); ! Fixed slope for all groups
    y ON m (bw); ! Fixed slope for all groups
    y ON x;  ! direct effect
%BETWEEN%
    x m y;
    m ON x (ab);
    y ON m (bb);
    y ON x;
MODEL CONSTRAINT:
    NEW(indb indw);
    indw=aw*bw;
    indb=ab*bb;

OUTPUT: TECH1 TECH8 CINTERVAL;
```

Only one value (aw and bw) is given to within-group first-stage and second-stage effect for all groups.

Separate the within effect from the between group effect.

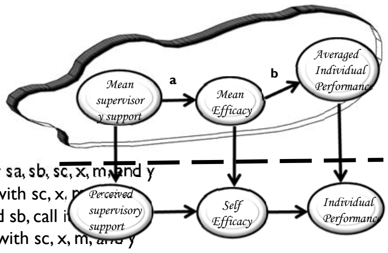
Calculate the within group mediation and between group mediation separately.

No bootstrapping statement



66

1-1-1 model (MSEM, random slope)

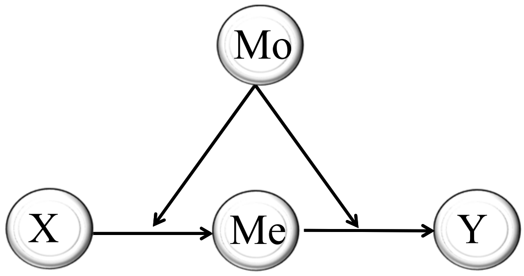



```
MODEL:
%WITHIN%
  sa | m ON x;
  sb | y ON m;
  sc | y ON x;

%BETWEEN%
sa sb sc x m y; ! estimate Level-2 (residual) variances for sa, sb, sc, x, m, and y
sa WITH sc x m y; ! estimate Level-2 covariances of sa with sc, x, m
sa WITH sb(cab); ! estimate Level-2 covariance of sa and sb, call it
sb WITH sc x m y; ! estimate Level-2 covariances of sb with sc, x, m, and y
sc WITH x m y; ! estimate Level-2 covariances of sc with x, m, and y
m ON x(ab); ! regress m on x, call the slope "ab"; ab = contextual effect, not the Between slope
y ON m(bb); ! regress y on m, call the slope "bb"; bb = contextual effect, not the Betweeen slope
y ON x; ! regress y on x
[sa](aw); ! estimate the mean of sa, call it "aw"
[sb](bw); ! estimate the mean of sb, call it "bw"

MODEL CONSTRAINT: ! section for computing indirect effects
NEW(a b indb indw); ! name the indirect effects
a=aw+ab; ! compute Between a path
b=bw+bb; ! compute Between b path
indw=aw*bw+cab; ! compute the Within indirect effect
indb=a*b; ! compute the Between indirect effect
```

Separate between and within mediation

67 Source: http://www.quantpsy.org/pubs/syntax_appendix_081311.pdf

Moderated Mediation



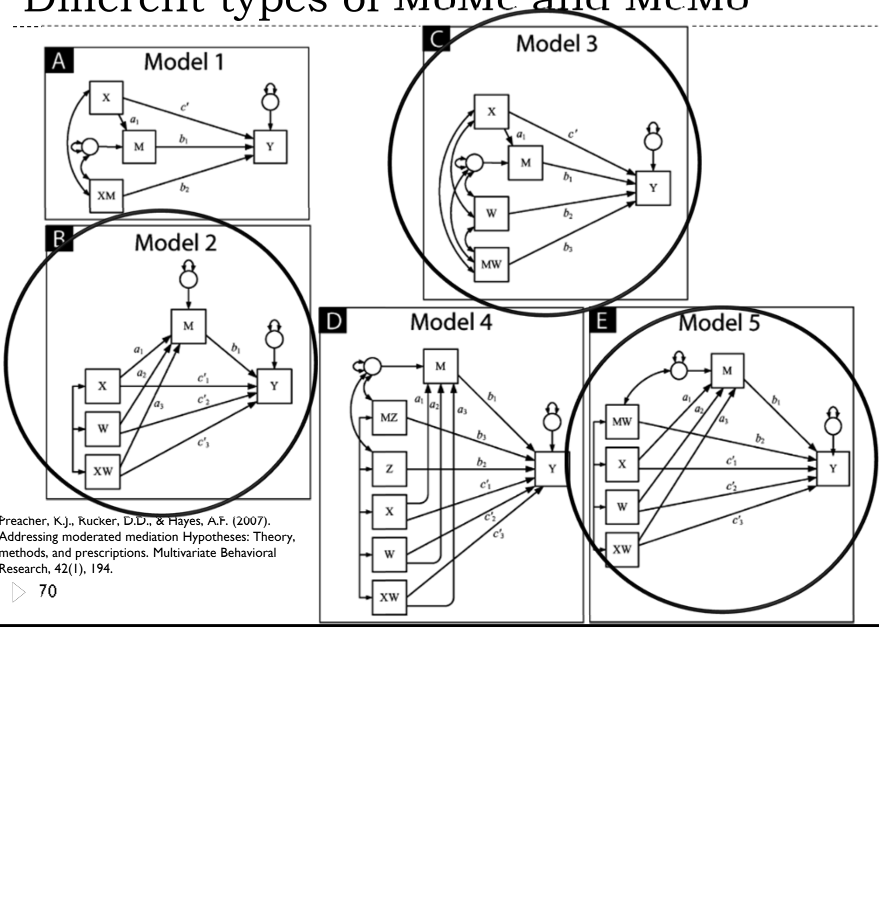
68

Combining Moderation and Mediation

- Muller, D., Judd, C.M., Yzerbyt, V.Y. (2005). When moderation is mediated and mediation is moderated. Journal of Personality and Social Psychology, 89(6), 852-863.
- Edwards, J.R., & Lambert, L.S. (2007). Methods for integrating moderating and mediation: A general analytical framework using moderated path analysis. Psychological Methods, 12, 1-22.
- Preacher, K.J., Rucker, D.D., & Hayes, A.F. (2007). Addressing moderated mediation hypotheses: Theory, methods, and prescriptions. Multivariate Behavioral Research, 42(1), 185-227.
- Preacher, K.J., & Hayes, A.F. (2008). Asymptotic and resampling strategies for assessing and comparing indirect effects in multiple mediator models. Behavioral Research Methods, 40(3), 879-891.
- Zhang, Z., Zyphur, M.J., & Preacher, K.J. (2009). Testing multilevel mediation using hierarchical linear models: Problems and solutions. Organizational Research Methods, 12(4), 695-719.
- Preacher, K.J., Zyphur, M.J., Zhang, Z. (2010). A general multilevel SEM framework for assessing multilevel mediation. Psychological Methods, 15(3), 209-233.
- Hayes, A.F. & Preacher, K.J. (2010). Quantifying and testing indirect effects in simple mediation models when the constituent paths are nonlinear. Multivariate Behavioral Research, 45, 627-660.

69

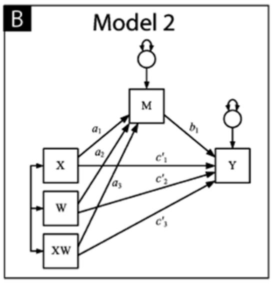
Different types of MoMe and MeMo

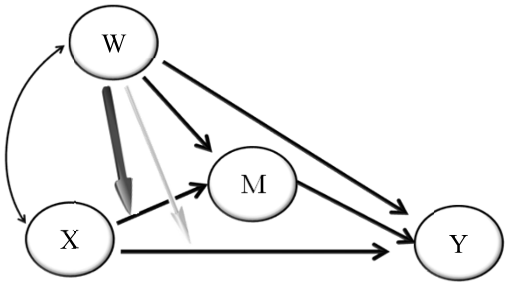


Preacher, K.J., Rucker, D.D., & Hayes, A.F. (2007). Addressing moderated mediation Hypotheses: Theory, methods, and prescriptions. Multivariate Behavioral Research, 42(1), 194.

70

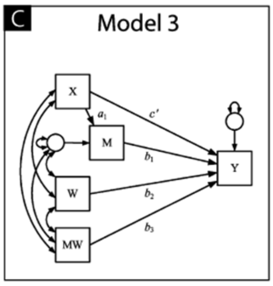
Model 2 of MoMe

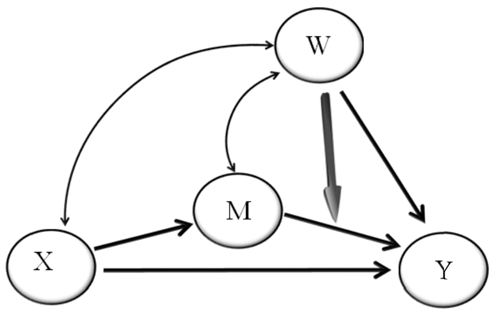




71

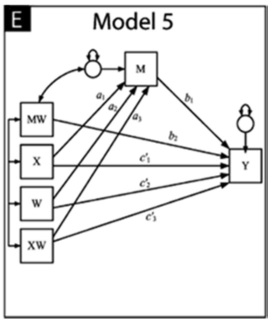
Model 3 of MoMe

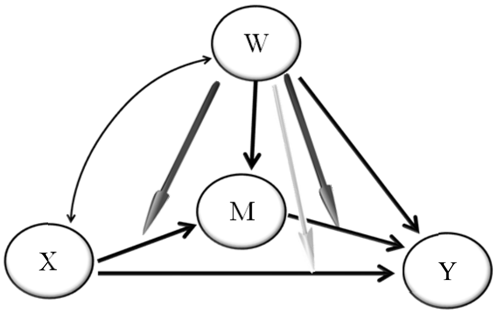




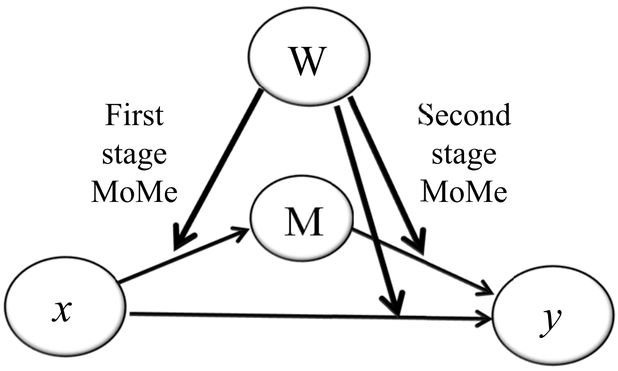
72

Model 5 of MoMe

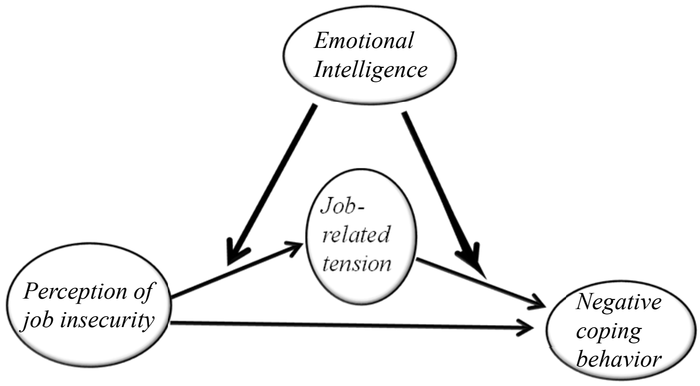




General case of Moderated Mediation



A model linking job insecurity to behavior

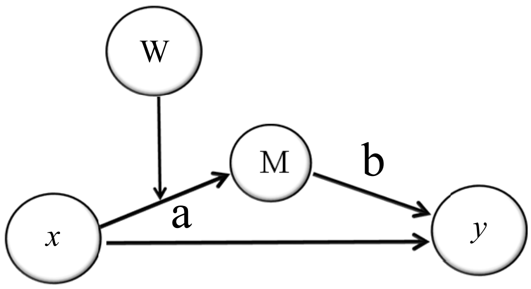


Jordan, P.J., Ashkanasy, N.M., & Hartel, C.E. (2002). Emotional intelligence as a moderator of emotional and behavioral reactions to job insecurity. Academy of Management Review, 27(3), 361-372.

75

First-stage Moderated Mediation

H_0 : mediating effect (W high) = mediating effect (W low)
H_1 : mediating effect (W high) > mediating effect (W low)

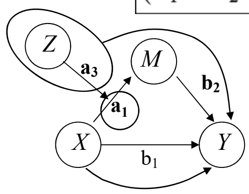


76

The structural model

$$\left(a_1+a_2Z\right)\,b_2$$

$$M=a_0+a_1X+a_2Z+a_3XZ+e_1$$
$$Y=b_0+b_1X+b_2M+e_2$$

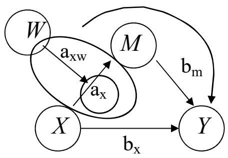


1st-stage moderation * 2nd-stage constant = moderated indirect effect

$$Y=\left[b_0+\left(a_0+a_2Z\right)\,b_2\right]+\left[\underbrace{b_1}_{\text{direct}}+\underbrace{\overbrace{\left(a_1+a_3Z\right)}^{\text{1st-stage moderation}}\,\overbrace{b_2}^{\text{2nd-stage constant}}}_{\text{Moderated indirect}}\right]X+\left(e_2+b_2e_1\right)$$

We do not know the size of the mediation effect.
The mediation effect depends on the value of Z.

The combined structural model



1st-stage moderation * 2nd-stage constant = moderated indirect effect

$$Y=\left[b_0+\left(a_0+a_wW\right)\,b_m\right]+\left[\underbrace{b_x}_{\text{direct}}+\underbrace{\overbrace{\left(a_x+a_{xw}W\right)}^{\text{1st-stage moderation}}\,\overbrace{b_m}^{\text{2nd-stage constant}}}_{\text{indirect}}\right]X+e$$

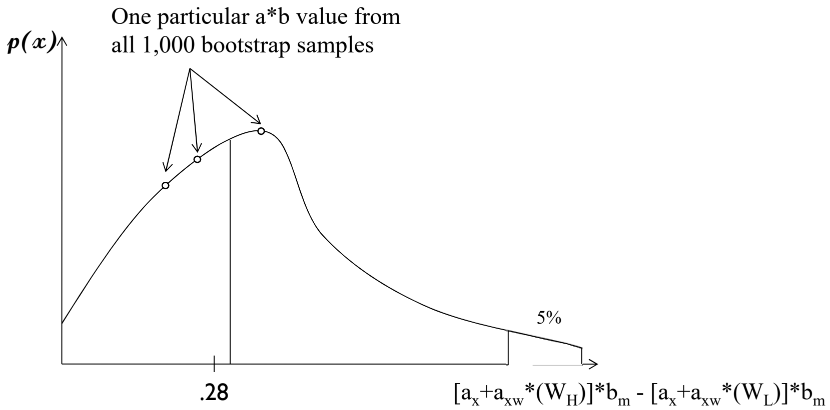
When W = Hi (+s) Indirect effect = $(a_x + a_{zw}*W_{Hi})b_m$

When W = Lo (- s) Indirect effect = $(a_x + a_{zw}*W_{Lo})b_m$

Need to use bootstrapping to test whether these point estimates are statistically different from zero.

Edwards, J.R. & Lambert, L.S. (2007). Methods for Integrating Moderation and Mediation: A General Analytical Framework Using Moderated Path Analysis. Psychological Methods, 2007, 12(1), 1-22.

Bootstrapping

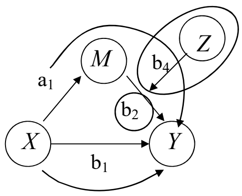


79

Second stage moderation

$$\boxed{a_1\left(b_2 + b_4 Z\right)}$$

$$M = a_0 + a_1 X + e_1$$
$$Y = b_0 + b_1 X + b_2 M + b_3 Z + b_4 MZ + e_{y10}$$

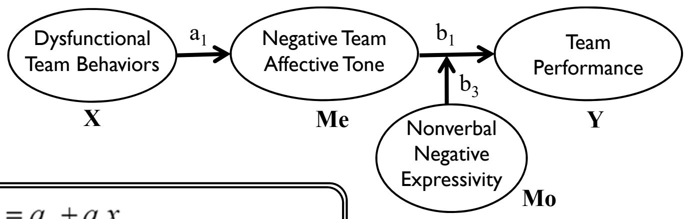


$\boxed{\text{1st-stage constant}} * \boxed{\text{2nd-stage moderation}}$ = moderated indirect effect

$$\boxed{Y = intercept + \left[\underset{\text{direct}}{b_1} + a_1\underbrace{\left(b_2 + b_4 Z\right)}_{\text{indirect}}\right]X + \left(b_3 + a_0 b_4\right)Z + error}$$

80

An example: Cole et al. (2008)



$$\boxed{\begin{aligned} Me &= a_0 + a_1 x \\ y &= b_0 + b_1 Me + b_2 Mo + b_3 MeMo \end{aligned}}$$

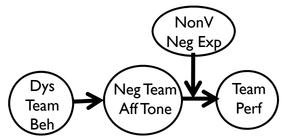
$$y = (b_0 + a_0 b_1) + \underbrace{\left[a_1\left(b_1 + b_3 Mo\right)\right]}_{\substack{\textit{Conditional indirect effect} \\ \textit{(moderated mediation effect)}}} x + (b_2 + a_0 b_3) Mo$$

81

An example: Cole et al. (2008)

$$Me = a_0 + a_1 x$$
$$y = b_0 + b_1 Me + b_2 Mo + b_3 MeMo$$

	b_0=-.01
a_0=.03	b_1=-.43
a_1=.62	b_2=-.02
	b_3=-.52



$$y = (b_0 + a_0 b_1) + \underbrace{\left[a_1\left(b_1 + b_3 Mo\right)\right] x}_{(.62)[(-.43)+(-.52)Mo]} + (b_2 + a_0 b_3) Mo$$

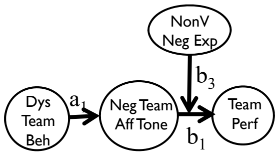
Table 3
Regression Results for Conditional Indirect Effect

Predictor	*B*	*SE*	*t*	*p*
Negative team affective tone				
Constant	0.03	0.06	0.42	.675
Dysfunctional team behavior	0.62	0.17	3.71	.001
Team performance				
Constant	−0.01	0.06	−0.19	.846
Negative team affective tone (NAT)	−0.43	0.13	−3.33	.002
Nonverbal negative expressivity (N-exp)	−0.02	0.12	−0.16	.877
NAT × N-exp	−0.52	0.24	−2.17	.035

82

An example: Cole et al. (2008)

$$Me = a_0 + a_1 x$$
$$y = b_0 + b_1 Me + b_2 Mo + b_3 MeMo$$



$$y = (b_0 + a_0 b_1) + \underbrace{\left[a_1 (b_1 + b_3 Mo) \right]}_{(.62)[(-.43)+(-.52)Mo]} x + (b_2 + a_0 b_3) Mo$$

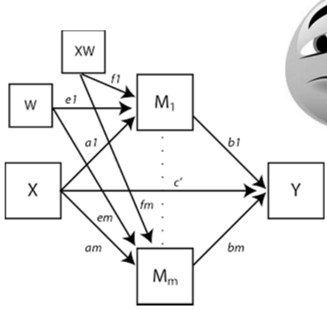
Table 3
Regression Results for Conditional Indirect Effect

Nonverbal negative expressivity	Boot indirect effect	Boot *SE*	Boot *z*	Boot *p*
Conditional indirect effect at N-exp = *M* ± 1 *SD*				
−1 *SD* (−0.55)	−0.07	0.15	−0.49	.626
M (−0.01)	−0.26	0.12	−2.17	.030
+1 *SD* (0.54)	−0.44	0.16	−2.74	.006

Mo=−1σ (-.55) Cond. Ind Eff = .62*[(-.43)+ (-.52)*(-.55)] = -.07 (n.s.)
Mo=mean (-.01) Cond. Ind Eff = .62*[(-.43)+ (-.52)*(-.01)]= -.26 (p<.05)
Mo=+1σ (+.55) Cond. Ind Eff = .62*[(-.43)+ (-.52)*(+.55)] = -.44 (p<.01)
Mod Med effect a_1 b_1 b_3

83

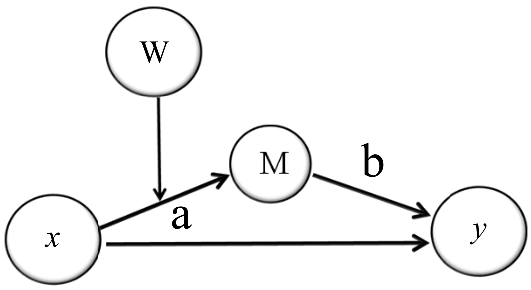
The programming of MoMe



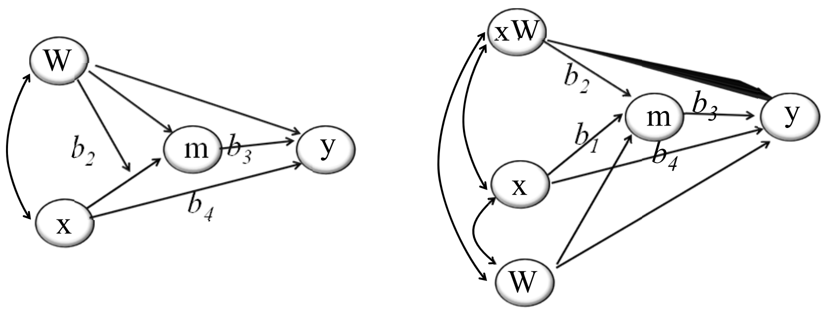
84

First-stage Moderated Mediation

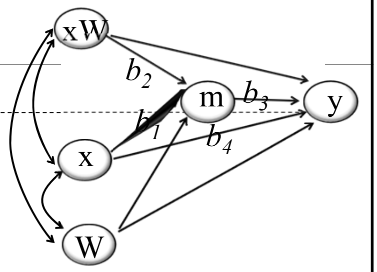
H_0 : mediating effect (W high) = mediating effect (W low)
H_1 : mediating effect (W high) > mediating effect (W low)



Single level first stage MoMe



The logic



$$m = a_0 + b_1 x + a_1 W + b_2 xW + \varepsilon_1 \quad (1)$$
$$y = a_2 + b_5 x + a_3 W + a_4 xW + b_3 m + \varepsilon_2 \quad (2)$$

From (1), we have $m = a_0 + a_1 W + \left(b_1 + b_2 W\right) x$

<u>If we ignore the intercept</u>, when W is high (1 SD above mean) and low (1 SD below mean), the effect of x on m are

$b_1 + b_2 W_h$ and $b_1 + b_2 W_l$

The indirect (mediation) effect of x to y through m when W is high versus low is $\left[\left(b_1 + b_2 W_h\right) * b_3\right] - \left[\left(b_1 + b_2 W_l\right) * b_3\right]$

Mplus program

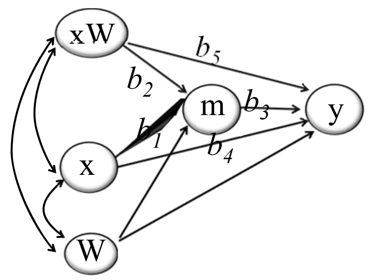
TITLE: mono-level first stage moderated mediation

DEFINE: xw=(x - 3.1163)*(w1 - 3.2809);
CENTER x m w1 (GRANDMEAN);
DATA:FILE IS example 1.dat;
VARIABLE:NAMES ARE x w m y;
USEVARIABLES ARE x m w y xw;
ANALYSIS:BOOTSTRAP=2000;

newly defined variables should appear at the end of the list

刘东，张震，汪默（2012）被调节的中介和被中介的调节：理论建构与模型检验。见 陈晓萍，徐淑英，樊景立主编。组织与管理研究的实证方法（第二版），北京大学出版社，553-590页。

Mplus program

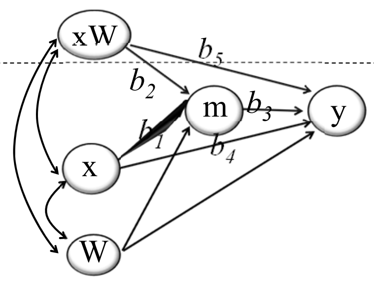



```
MODEL:
m ON  x (b1)
        w
        xw (b2);
  y ON   m (b3)
        x (b4)
        w
        xw (b5);
```

$$m = a_0 + b_1 x + a_1 w + b_2 xw$$
$$y = a_2 + b_3 m + b_4 x + a_3 w + b_5 xw$$

89

Mplus program



Note: W_H = .8552 ; W_L = -.8552

Indirect effect

```
MODEL CONSTRAINT:
NEW(ind_h  ind_l  ind_d);
ind_h=(b1+b2*0.8552)*b3;
ind_l=(b1+b2*(-0.8552))*b3;
ind_d=ind_h - ind_l;
```

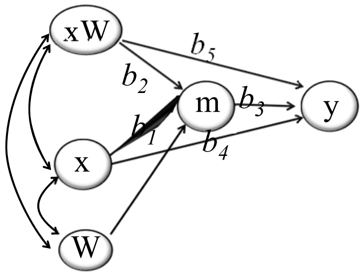
$$m = a_0 + b_1 x + a_1 w + b_2 xw$$
$$y = a_2 + b_3 m + b_4 x + a_3 w + b_5 xw$$

```
OUTPUT:
SAMPSTAT;
CINTERVAL(BCBOOTSTRAP);
```

$$\left[\left(b_1 + b_2 W_h\right) * b_3\right] - \left[\left(b_1 + b_2 W_l\right) * b_3\right]$$

90

Mplus Output



MODEL RESULTS

		Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
	M ON				
b_1	X	0.721	0.042	17.091	0.000
	W	0.713	0.041	17.506	0.000
b_2	XW	0.303	0.052	5.844	0.000
	Y ON				
b_3	M	0.785	0.089	8.809	0.000
b_4	X	-0.100	0.081	-1.231	0.218
	W	-0.167	0.080	-2.087	0.037
b_5	XW	-0.045	0.064	-0.704	0.482
	Intercepts				
	M	0.045	0.036	1.272	0.204
	Y	2.166	0.045	48.271	0.000
	Residual Variances				
	M	0.251	0.027	9.333	0.000
	Y	0.350	0.039	9.040	0.000
	New/Additional Parameters				
	IND_H	0.769	0.098	7.851	0.000
	IND_L	0.363	0.071	5.126	0.000
	IND_D	0.406	0.079	5.117	0.000

$$\left[(b_1 + b_2W_h) * b_3\right] - \left[(b_1 + b_2W_l) * b_3\right] \longrightarrow \text{IND\_D}$$

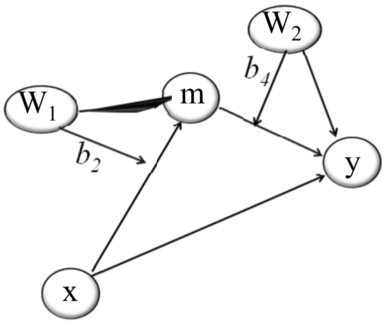
Output: Bootstrapping results

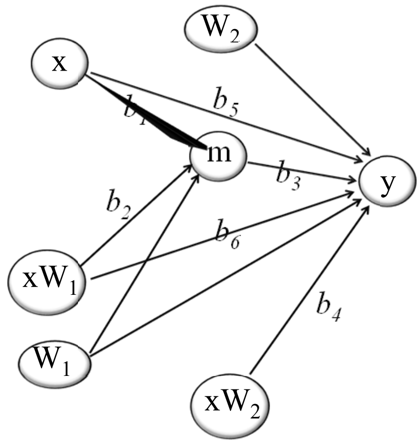
CONFIDENCE INTERVALS OF MODEL RESULTS

	Lower .5%	Lower 2.5%	Lower 5%	Estimate	Upper 5%	Upper 2.5%	Upper .5%
M ON							
X	0.614	0.639	0.653	0.721	0.790	0.803	0.842
W	0.616	0.632	0.646	0.713	0.781	0.792	0.815
XW	0.166	0.200	0.217	0.303	0.387	0.400	0.447
Y ON							
M	0.547	0.611	0.636	0.785	0.932	0.960	1.014
X	-0.306	-0.263	-0.235	-0.100	0.028	0.049	0.095
W	-0.382	-0.321	-0.300	-0.167	-0.037	-0.011	0.044
XW	-0.209	-0.171	-0.152	-0.045	0.060	0.084	0.122
Intercepts							
M	-0.046	-0.024	-0.013	0.045	0.105	0.114	0.142
Y	2.054	2.079	2.093	2.166	2.238	2.254	2.285
Residual Variances							
M	0.193	0.206	0.213	0.251	0.303	0.313	0.327
Y	0.267	0.288	0.298	0.350	0.429	0.447	0.472
New/Additional Parameters							
IND_H	0.527	0.592	0.619	0.769	0.953	0.980	1.031
IND_L	0.207	0.240	0.259	0.363	0.494	0.521	0.569
IND_D	0.230	0.269	0.293	0.406	0.559	0.580	0.654

$$\left[(b_1 + b_2W_h) * b_3\right] - \left[(b_1 + b_2W_l) * b_3\right] \longrightarrow \text{IND\_D}$$

Single level first and second stage MoMe

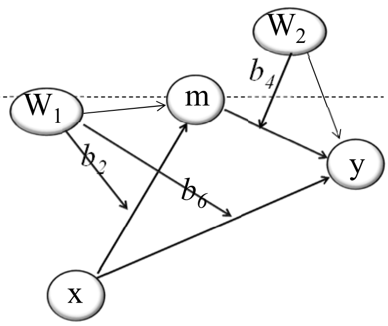




Note: all exogenous variables are correlated by default

93

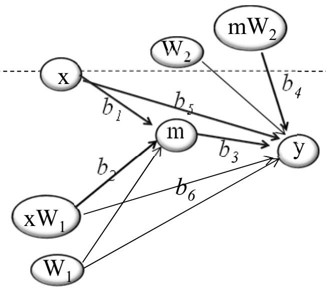
The logic



- In this case, the effect of x on m is contingent on W_1; the effect m on y is contingent on W_2.
- There should be four cases of $W_{1h}W_{2h}$, $W_{1h}W_{2l}$, $W_{1l}W_{2h}$ and $W_{1l}W_{2l}$. ; high & low defined by mean + σ or – σ
- For simplicity, we only compare two extreme cases of indirect effect, $W_{1h}W_{2h}$ and $W_{1l}W_{2l}$, i.e., Hi-Hi and Lo-Lo.

94

The logic



Note: (W_1 high & W_2 high) – (W_1 low & W_2 low)

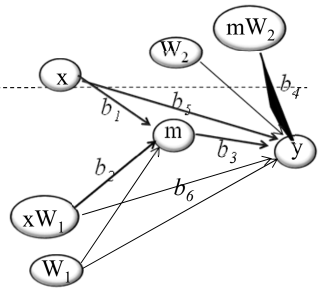
$$m = a_0 + b_1 x + a_1 W_1 + b_2 x W_1 + \varepsilon_1$$

$$y = a_2 + b_5 x + a_3 W_2 + b_4 m W_2 + b_3 m + a_4 W_1 + b_6 x W_1 + \varepsilon_2$$

- Indirect effect of x on y through m when W_1 is high and W_2 is high is $(b_1+b_2W_{1h})*(b_3+b_4W_{2h})$;
- Indirect effect of x on y through m when W_1 is low and W_2 is low is $(b_1+b_2W_{1l})*(b_3+b_4W_{2l})$;
- Differences in indirect effect when the W_1 and W_2 is high and low is: $\left(b_1 + b_2 W_{1h}\right)\left(b_3 + b_4 W_{2h}\right) - \left(b_1 + b_2 W_{1l}\right)\left(b_3 + b_4 W_{2l}\right)$

95

Mplus program



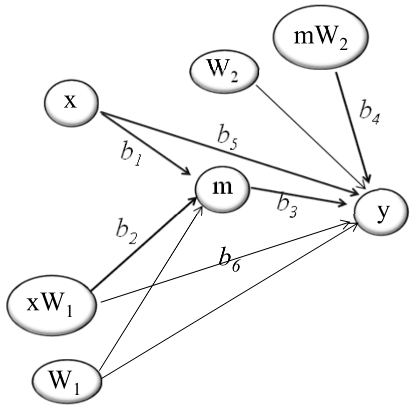
DEFINE:
xw1=(x - 3.1163)*(w1 - 3.2809);
mw2=(m - 2.7560)*(w2 - 3.1505);

CENTER x m w1 w2 (GRANDMEAN);
DATA:FILE IS example 1.dat;
VARIABLE:NAMES ARE x w1 m w2 y;
USEVARIABLES ARE x m w1 w2 y **xw1 mw2**;
ANALYSIS:BOOTSTRAP=2000;

96

Mplus program

```
MODEL:
 m ON  x (b1)
       w1
       xw1 (b2);
y ON  m (b3)
       w2
       mw2 (b4)
       x (b5)
       w1
       xw1 (b6);
```

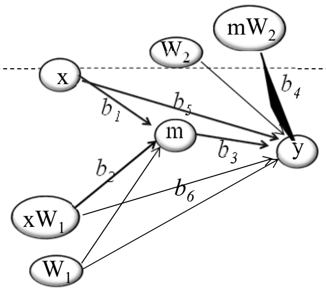


97

Mplus program

Note 1: SD W_1 = .8552; SD of W_2 = .8819

```
MODEL CONSTRAINT:
NEW(ind_hh  ind_ll  ind_d);
ind_hh=(b1+b2*0.8552)*(b3+b4*.8819);
ind_ll=(b1+b2*(-0.8552))*(b3+b4*(-.8819));
ind_d=ind_hh - ind_ll;

OUTPUT:
SAMPSTAT;
CINTERVAL(BCBOOTSTRAP);
```



Second stage MoMe effect

$$\boxed{(b_1 + b_2 W_{1h})(b_3 + b_4 W_{2h}) - (b_1 + b_2 W_{1l})(b_3 + b_4 W_{2l})}$$

First stage MoMe effect

*Note 2: (First stage Hi * Second stage Hi) – (First stage Lo * Second stage Lo)*

98

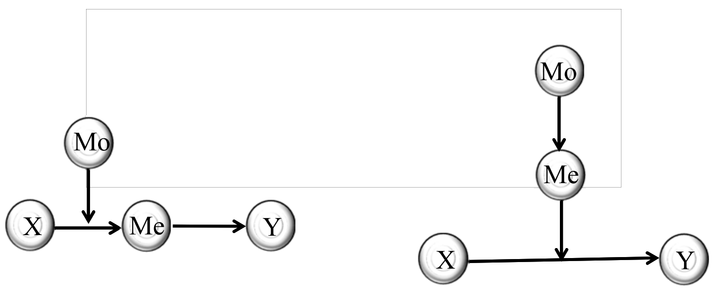
Output

CONFIDENCE INTERVALS OF MODEL RESULTS

	Lower .5%	Lower 2.5%	Lower 5%	Estimate	Upper 5%	Upper 2.5%	Upper .5%
New/Additional Parameters							
IND_HH	0.781	0.818	0.839	0.945	1.048	1.069	1.117
IND_LL	0.157	0.178	0.190	0.255	0.328	0.342	0.376
→ **IND_D**	0.502	0.544	0.571	0.690	0.812	0.835	0.884

$$\text{IND\_D} = (b_1 + b_2 W_{1h})(b_3 + b_4 W_{2h}) - (b_1 + b_2 W_{1l})(b_3 + b_4 W_{2l})$$

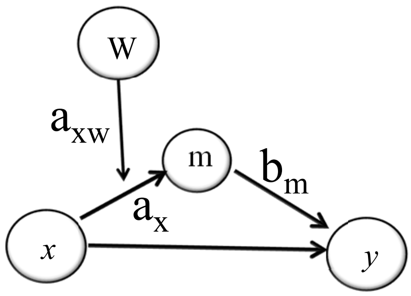
Mediated Moderation



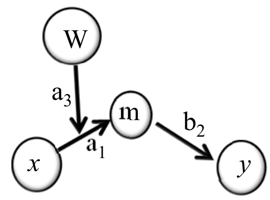
MoMe versus MeMo

Moderated Mediation

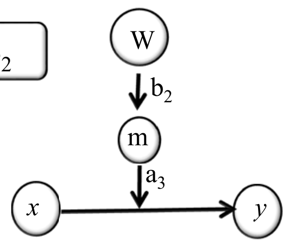
$(a_x + a_{wx}*W_{Hi})b_m - (a_x + a_{wx}*W_{Lo})b_m$



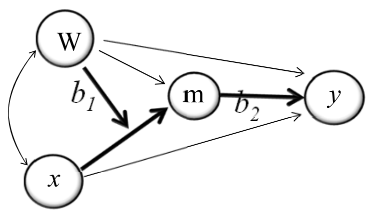
Mediated Moderation

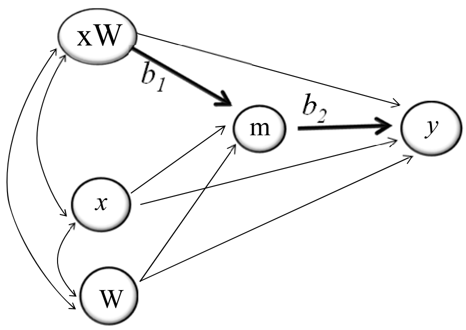


a_3*b_2



MeMo: Type I



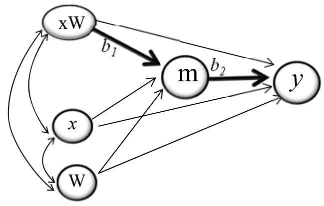


Note: The structural model is the same as first stage MoMe.
The only difference is the logic used to test it.

刘东，张震，汪默（2012）被调节的中介和被中介的调节：理论建构与模型检验。见 陈晓萍，徐淑英，樊景立主编。组织与管理研究的实证方法（第二版），北京大学出版社，553-590页。

The logic

$$m = a_0 + a_1x + a_2W + b_1xW + \varepsilon_1$$
$$y = a_3 + a_4x + a_5W + a_6mW + b_2m + \varepsilon_2$$

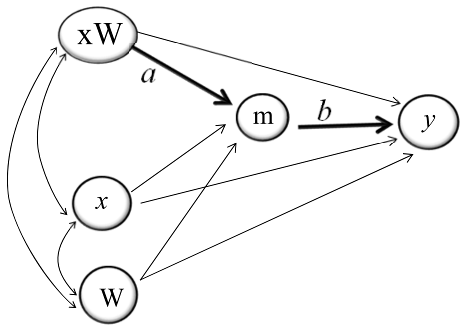


If one define mediation effect as the product of stage 1 effect and stage 2 effect, this model refers to the case of "the effect xW on m" times "the effect of m on y."

The effect size of this Type I MeMo model is, therefore, b_1b_2.

Mplus program

```
Title: mono-level type I mediated moderation
define: xw=(xo-3.1163)*(w-3.2809);
define: x=(xo-3.1163);
define: w=wo-3.2809;
data: file=example 1.dat;
variable: names=xo wo m y;
usevariable=m y xw x w;
analysis:
bootstrap=2000;

Model:
   m on x w;
   m on xw (a);
   y on x w xw;
   y on m (b);

Model constraint:
new(ind);
ind=a*b;

output:tech1
cinterval(bcbootstrap);
```



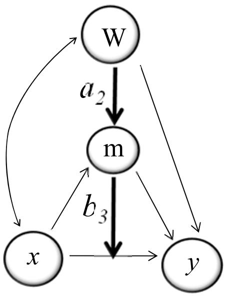
Output

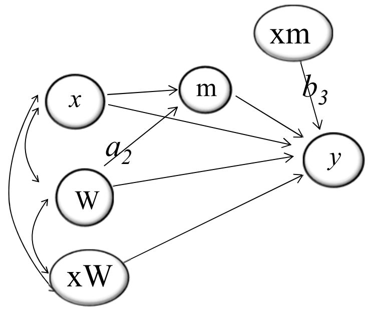
CONFIDENCE INTERVALS OF MODEL RESULTS

	Lower .5%	Lower 2.5%	Lower 5%	Estimate	Upper 5%	Upper 2.5%	Upper .5%
M ON							
X	0.614	0.639	0.653	0.721	0.790	0.803	0.842
W	0.616	0.632	0.646	0.713	0.781	0.792	0.815
XW	0.166	0.200	0.217	0.303	0.387	0.400	0.447
Y ON							
X	-0.305	-0.263	-0.235	-0.100	0.027	0.049	0.095
W	-0.382	-0.321	-0.300	-0.167	-0.037	-0.011	0.044
XW	-0.209	-0.171	-0.153	-0.045	0.059	0.084	0.122
M	0.547	0.611	0.636	0.785	0.932	0.960	1.014
New/Additional Parameters							
IND	0.135	0.158	0.172	0.238	0.327	0.339	0.382

105

MeMo: Type II





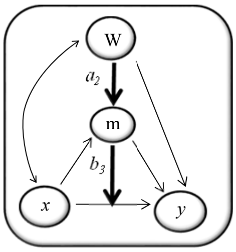
This model means that W moderates the x→y relationship, and this relationship is mediated by m.

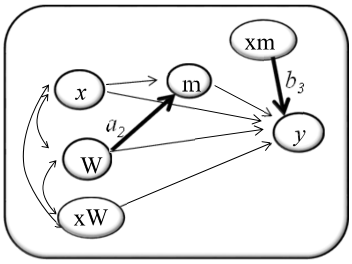
$$y = b_0 + b_1x + b_2m + b_3xm + b_4W + b_5xw$$
$$m = a_0 + a_1x + a_2w$$

106

Mplus program

```
TITLE; A mono-level Type II MeMo model;
DEFINE:
xw = (x - 3.0) * (w - 4.0);
xm = (x - 3.0) * (m - 2.5);
DATA; FILE IS example. txt;
VARIABLE: NAMES ARE x m w y;
USEVARIABLES ARE x m w y xw xm;
CENTERING IS GRANDMEAN (x m w) ;
ANALYSIS: BOOTSTRAP = 2000;
MODEL:
  m ON w ( a2)
  x;
y ON xm ( b3)
  m x w xw;
MODEL CONSTRAINT:
NEW (ind) ;
ind = a2 * b3;
OUTPUT:
SAMPSTAT;
CINTERVAL( BCBOOTSTRAP) ;
```



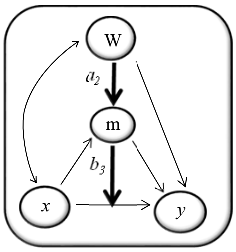


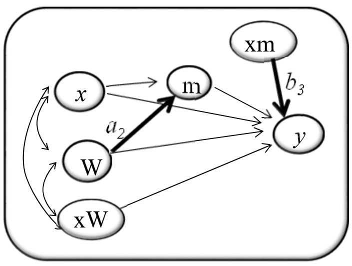
107

Mplus program

$$y = b_0 + b_1x + b_2m + b_3xm + b_4W + b_5xw$$
$$m = a_0 + a_1x + a_2w$$

```
MODEL:
  m ON w ( a2)
  x;
  y ON xm ( b3)
  m x w xw;

MODEL CONSTRAINT:
NEW (ind) ;
ind = a2 * b3;
```





108

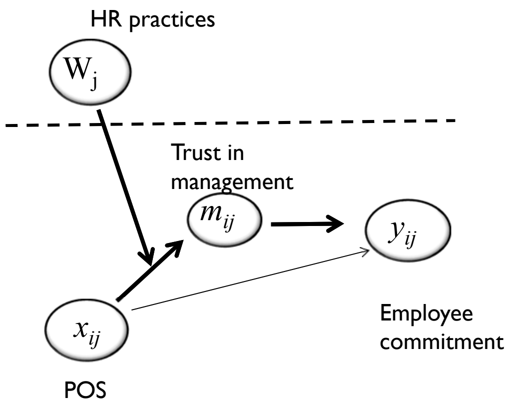
Generalization to …

Cross-level MoMe and MeMo

刘东，张震，汪默（）被调节的中介和被中介的调节：理论建构与模型检验。见 陈晓萍，徐淑英，樊景立主编。组织与管理研究的实证方法（第二版），北京大学出版社，553-590页。

109

Two-level first stage MoMe



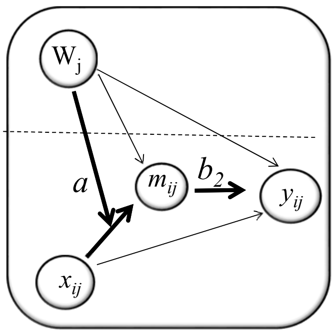
Whitener, E.M. (2001). Do "high commitment" human resource practices affect employee commitment? A cross-level analysis using hierarchical linear modeling. Journal of Management, 27, 515-535.

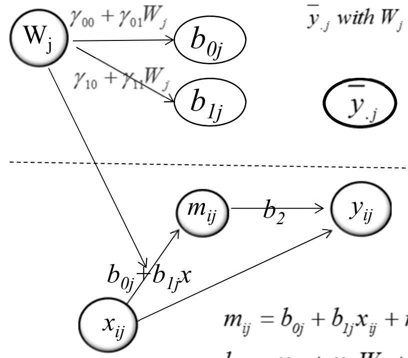
110

Two-level first stage MoMe

Correlates:
b_{0j} *with* b_{1j}
$\bar{y}_{.j}$ *with* b_{0j}
$\bar{y}_{.j}$ *with* b_{1j}
$\bar{y}_{.j}$ *with* W_j

Note:
W_j means it is a level 2 variable
x_{ij} means it is a level 1 variable





$$m_{ij} = b_{0j} + b_{1j}x_{ij} + r_{ij}$$
$$b_{0j} = \gamma_{00} + \gamma_{01}W_j + u_{0j}$$
$$b_{1j} = \gamma_{10} + \gamma_{11}W_j + u_{1j}$$

Effect size $= (\gamma_{10} + \gamma_{11}W_j)\, b_2$

111

Why do we need these correlates?

- The Mplus program will automatically correlate all exogenous variables. However, one needs to specify what level 2 endogenous variables created in our model should be correlated. For example, the random intercept (b_{0j}), the random slope (b_{1j}), group mean of y ($\bar{y}_{.j}$) are created in our model.
- In our model, these three variables and W_j are all exogenous at level 2. We allow them to correlate:

HLM terminology	Mplus terminology
b_{0j} *with* b_{1j}	m with S
$\bar{y}_{.j}$ *with* b_{0j}	y with m
$\bar{y}_{.j}$ *with* b_{1j}	y with S
$\bar{y}_{.j}$ *with* W_j	y with W

Correlates:
b_{0j} *with* b_{1j}
$\bar{y}_{.j}$ *with* b_{0j}
$\bar{y}_{.j}$ *with* b_{1j}
$\bar{y}_{.j}$ *with* W_j

112

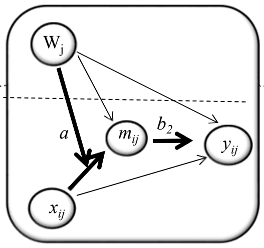
Mplus program

TITLE: A two-level first-stage MoMe;

DATA; FILE IS example.txt;
VARIABLE: NAMES ARE x m w y cluster;
USE VARIABLES ARE x m w y;
CENTERING IS GRANDMEAN(w) ;
CENTERING IS GROUPMEAN(x) ;
CLUSTER = cluster;
WITHIN = x;
BETWEEN = w;

ANALYSIS: TYPE = TWOLEYEL RANDOM;

Mplus program



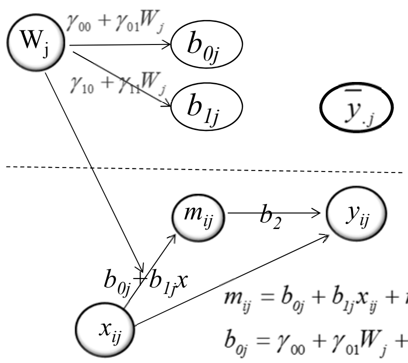
MODEL:
% WITHIN %
S | m on x;
y on m (b2)
x;
% BETWEEN%
S on w (al) ; $\longleftrightarrow \gamma_{11}$
[S] (a0) ; $\longleftrightarrow \gamma_{10}$
m on w;
m with S;
y with m;
y with S;
y with w;

The effect of x on m as a slope

Effect of m on y as called it b_2

x on y

Correlates:
b_{0j} *with* b_{1j}
$\bar{y}_{.j}$ *with* b_{0j}
$\bar{y}_{.j}$ *with* b_{1j}
$\bar{y}_{.j}$ *with* w_j



$$m_{ij} = b_{0j} + b_{1j} x_{ij} + r_{ij}$$
$$b_{0j} = \gamma_{00} + \gamma_{01} W_j + u_{0j}$$
$$b_{1j} = \gamma_{10} + \gamma_{11} W_j + u_{1j}$$

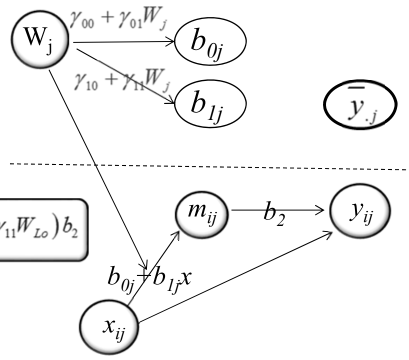
Mplus program

MODEL CONSTRAINT:

NEW (ind_h ind_l) ;
ind_h = (a0+al*(.85))*b2; $(\gamma_{10}+\gamma_{11}W_{Hi})b_2$
ind_l = (a0 + al*(-.85))*b2; $(\gamma_{10}+\gamma_{11}W_{Lo})b_2$

NEW (diff) ;
Diff = ind_h - ind_l; $\boxed{(\gamma_{10}+\gamma_{11}W_{Hi})b_2-(\gamma_{10}+\gamma_{11}W_{Lo})b_2}$

OUTPUT:
SAMPSTAT;
CINTERVAL;



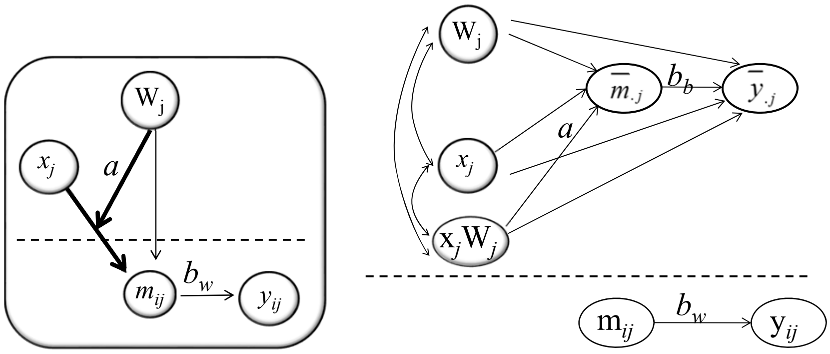
No bootstrapping command because Mplus cannot do bootstrapping for multi-level models at the moment.

115

Cross-level Type I MeMo

116

Two-level Type I MeMo model



Since we cannot multiple a second level $(x_j w_j \rightarrow \overline{m}_{.j})$ effect by a first level $m_{ij} \rightarrow y_{ij}$, all the mediation has to be happened at the group level. The mediating effect size should be $\boxed{(x_j w_j \rightarrow \overline{m}_{.j})(\overline{m}_{.j} \rightarrow \overline{y}_{.j})}$

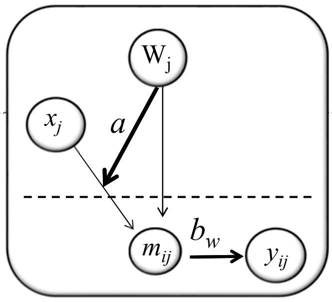
117

Mplus program

```
TITLE: A two-level Type I MeMo model,
DATA: FILE IS example.txt;
DEFINE:
xw = (x -3.0) * (w - 4.0) ;

VARIABLE: NAMES ARE x m w y cluster;
USEVAR1ABLES ARE x m w y xw;
CENTERING IS GRANDMEAN(x w) ;
CLUSTER = cluster;
BETWEEN = x w xw;

ANALYSIS: TYPE = TWOLEVEL;
```

118

Mplus program

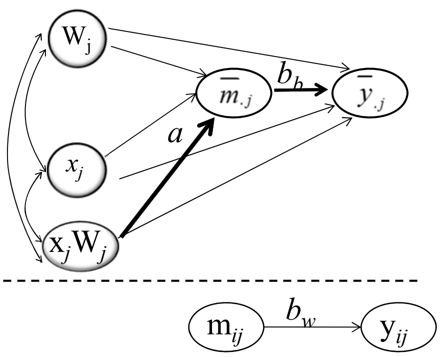



```
MODEL:

 % WITHIN %
  y on m (bw)

% BETWEEN%
  m on x w
    xw (a) ;
  y on m (bb)
    x w xw;

MODEL CONSTRAINT;
NEW (ind) ;
  ind = a * bb;

OUTPUT:
SAMPSTAT;
CINTERVAL;
```

This is for control only. We will not use it.

x_jW_j on $\bar{m}_{.j}$ at the group level

$\bar{m}_{.j}$ on $\bar{y}_{.j}$ at the group level

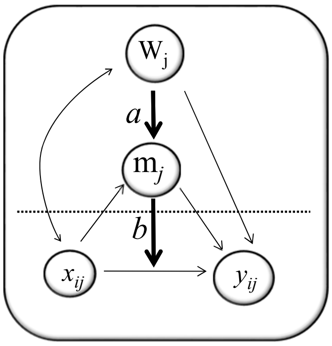


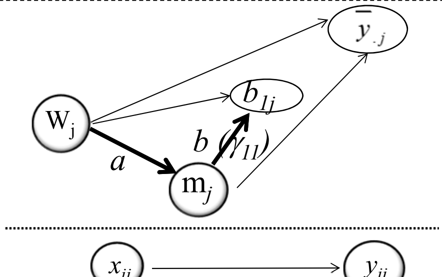
119

Cross-level Type II MeMo

120

Two-level Type II MeMo model





$$y_{ij} = b_{0j} + b_{1j}x_{ij} + r_{ij}$$
$$b_{0j} = \gamma_{00} + \gamma_{01}m_j + u_{0j}$$
$$b_{1j} = \gamma_{10} + \gamma_{11}m_j + u_{1j}$$

- While we model both the W_j to $x_{ij} \rightarrow y_{ij}$ cross-level interaction and the m_j to $x_{ij} \rightarrow y_{ij}$ cross-level interaction, the latter is of real interest to us because it is the most proximal moderator that forms the second half of the mediation effect (*b* effect).
- Since there is not a specified path from b_{1j} to $\bar{y}_{.j}$, we allow them to correlate at level 2.

121

Mplus program

```
TITLE : A two-level Type II MeMo;

DATA: FILE IS example.txt;
VARIABLE: NAMES ARE x m w y cluster;
USEVARIABLES ARE x m w y;
CENTERING IS GRANDMEAN(w m) ;
CENTERING IS GROUPMEAN(x) ;

CLUSTER = cluster;
WITHIN = x;
BETWEEN = w m;
```

122

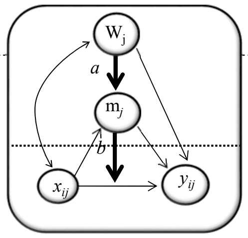
Mplus program

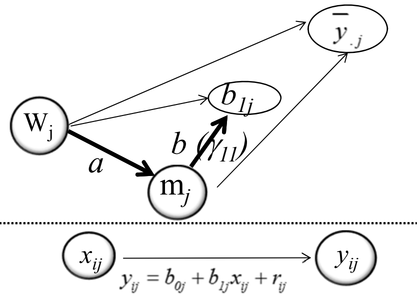
```
ANALYSIS:
TYPE =TWOLEVEL RANDOM;
% WITHIN %
  S | y on x;
% BETWEEN%
  m on w (a) ;
  S on m (b)
    w;
y on m w;
y with S;

MODEL CONSTRAINT;
NEW (ind) ;
  ind = a * b;
OUTPUT;
SAMPSTAT;
CINTERVAL;
```

Within group slope of x→y

Effect of m on slope, called it b (γ_{11})

Allow random slope to correlate with $\bar{y}_{.j}$

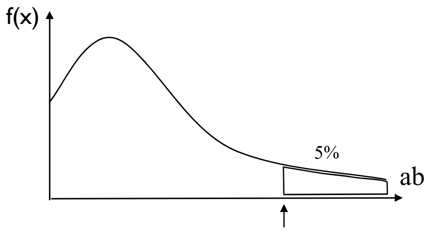




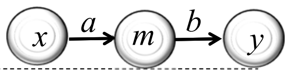
$$y_{ij} = b_{0j} + b_{1j}x_{ij} + r_{ij}$$
$$b_{0j} = \gamma_{00} + \gamma_{01}m_j + u_{0j}$$
$$b_{1j} = \gamma_{10} + \gamma_{11}m_j + u_{1j}$$

Bootstrapping in cross level analyses

N_1: $(ab)_1$
N_2: $(ab)_2$
……
……
……
N_{10000}: $(ab)_{10000}$



Two types of bootstrapping



Sampling bootstrapping

Original Sample	x_1	x_2	x_3
1	3	4	6
2	3	2	5
3	4	4	3
4	5	6	4
5	1	2	1
6	2	3	1
7	4	3	2
8	1	3	4
9	3	3	2
10	1	1	2
11			

$r_{12} = .76 \quad r_{23} = .38$

$ab = .29$

Sample one	x_1	x_2	x_3
1	3	4	6
2	3	4	6
3	3	4	6
4	1	1	2
5	2	3	1
6	4	4	3
7	4	3	2
8	5	6	4
9	4	4	3
10	3	2	5

∘ ∘ ∘

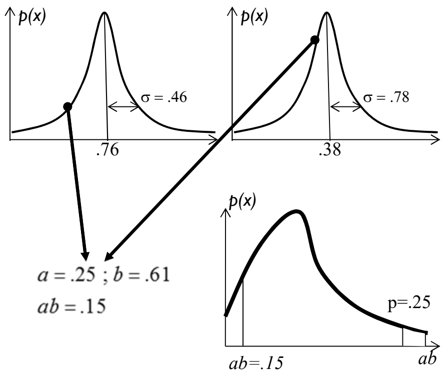
$r_{12} = .80 \quad r_{23} = .35$

$ab = .28$

parametric bootstrapping

$r_{12} = a = .76 \quad r_{23} = b = .38$

$SE = .46 \quad SE = .78$

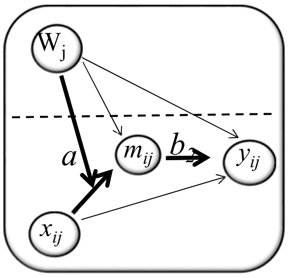


125

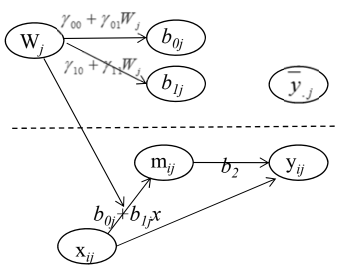
R code for parametric bootstrapping

Cross level Moderated Mediation (MoMe)

```
##################################################################
# a0 is the conditional mean of the random slope effect between X and M        #
# a0std is the standard error of a0                                            #
# a1 is the predictive effect of W on the random slope effect between X and M #
# a1std is the standard error of a1                                            #
# b is the fixed effect of M on Y                                              #
# bstd is the standard error of b                                              #
# "rep=20000 defines the number of resampling to be 20000                #
#"conf =95" defines that 95% CI will be used.                                   #
##################################################################
```



a0 = 0.979 γ_{10}
al = 0.050 γ_{11}
b = 0.540 b_2
a0std = 0. 051
alstd = 0. 058
bstd =0.029
Standard error of γ_{10}, γ_{11} & b_2
rep = 20000 Resample 20,000 times
conf = 95 Write the 95% bootstrap interval



126

R code for parametric bootstrapping

```
a0vec = rnorm( rep) * a0std + a0
alvec = rnorm (rep) * alstd + al
bvec = rnorm (rep) * bstd + b
```

Randomly draw from normal distributions of γ_{10}, γ_{11} & b_2 with specified mean and S.D.

```
amhvec = alvec * 0. 85 + a0vec
amlvec = alvec * (- 0. 85) + a0vec
```

For each γ_{10} & γ_{11} drawn, calculate ($\gamma_{10} + \gamma_{11} W_{jH}$) and ($\gamma_{10} + \gamma_{11} W_{jL}$)

```
abh = amhvec * bvec
abl = amlvec * bvec
d = abh – abl
```

$$\boxed{\left(\gamma_{10} + \gamma_{11} W_{jH}\right) b_2 - \left(\gamma_{10} + \gamma_{11} W_{jL}\right) b_2}$$

```
low = (1 - conf/100) /2
upp = ( 0 - conf/100)/2) + (conf/100)
```

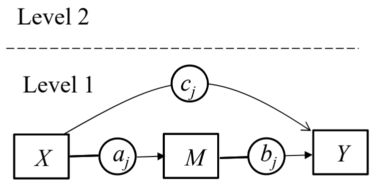
Draw the confidence interval

```
LL = quantile ( d, low )
UL = quantile ( d, upp )
LL4 = format ( LL, digits = 5 )
UL4 = format( UL,digits = 5)
hist( d, breaks = ‘ FD ’ , col = ‘ skyblue ’ , xlab = paste ( conf, ‘ % Confidence

Interval ’ , ‘ LL ’ , LL4 , ‘ UL ’ , UL4) , main = ‘ Distribution of Indirect Effect’ )
```

Plot the results

Parametric bootstrapping with correlated parameters

- 1-1-1 cross level model in Preacher, Zyphur & Zhang (2010). A general multilevel SEM framework for assessing multilevel mediation. Psychological Methods, 15(3), 209-233.



$$M_{ij} = a_0 + a_j X_{ij} + e_{Mij}$$
$$Y_{ij} = b_0 + b_j M_{ij} + c_j X_{ij} + e_{Yij}$$

a_j is the effect of X on M

b_j is the effect of M on Y conditional on X

Each level (j) would have a different value of a_j and b_j

However, the values of a_j and b_j *may be correlated.*

Heterogeneity in causal effects across levels

- The average of $a_j b_j$ is: $E(a_j b_j) = ab + \tau_{a_j b_j}$ (Goodman, 1960, p.712)

We nee to simulates a multivariate normal sampling distribution for these 3 estimates and uses the resulting pile of estimates to produce a sampling distribution of: $ab + \tau_{a_j b_j}$

Source: Bauer, Preacher and Gil, 2006

Generalization to …

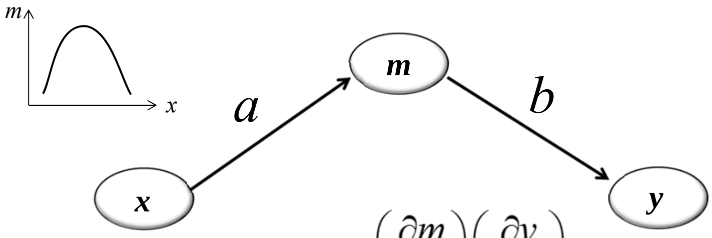
Curvilinear Mediation

130

Hayes, A.F. & Preacher, K.J. (2010) Quantifying and testing indirect effects in simple mediation models when the constituent paths are nonlinear. Multivariate Behavioral Research, 45(4), 627-660.

Curvilinear mediation

$$m = a_0 + a_1 x + a_2 x^2$$

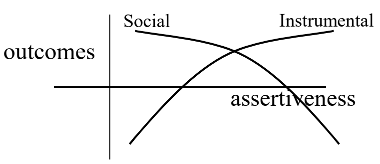


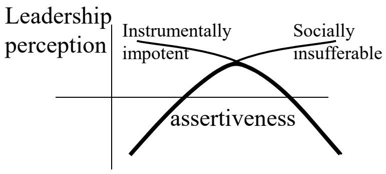
$$\theta = ab = \left(\frac{\partial m}{\partial x}\right)\left(\frac{\partial y}{\partial m}\right)$$

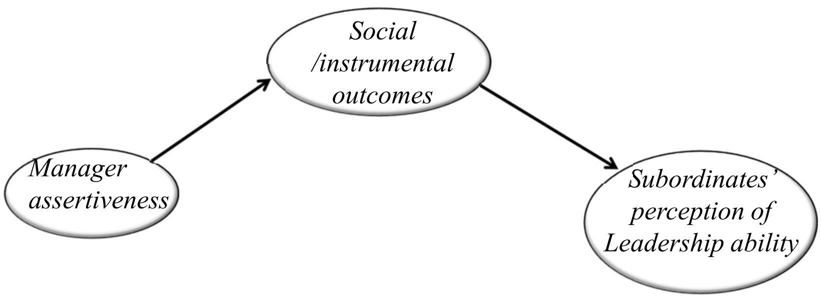
$$\theta = \left(\frac{\partial m}{\partial x}\right)\left(\frac{\partial y}{\partial m}\right) = \left(a_1 + 2a_2 x\right) b$$

131

An example





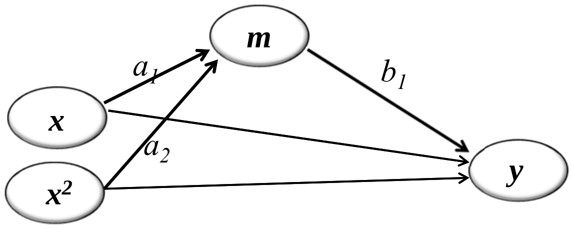


Ames, D.R. & Flynn, F.J. (2007) What breaks a leader: The curvilinear relation between assertiveness and leadership. Journal of Personality and Social Psychology, 92(2), 307-324.

132

Curvilinear mediation

$$m = a_0 + a_1 x + a_2 x^2$$



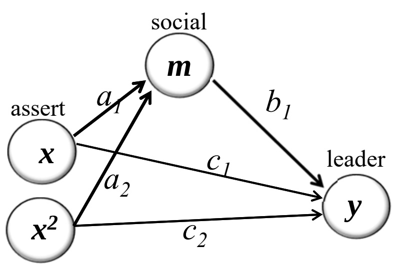
$$\boxed{\theta = (a_1 + 2a_2 x)\, b_1}$$

133

Mplus program

```
TITLE: Ames and Flynn (2007) example;
DATA: file is c:\ames.dat;
VARIABLE: names are x y m xsq;
usevariables are x y m xsq;

ANALYSIS:
bootstrap = 10000;

MODEL:
m on x (a1)         x→m
   xsq (a2);        x²→m
y on x (c1)         Linear direct effect
   xsq (c2)         Quadratic direct effect
   m (b1);          m→y
[m] (a0);           This is the intercept when x→m
```

$$\boxed{\begin{aligned} m &= a_0 + a_1 x + a_2 x^2 \\ y &= b_0 + b_1 m \end{aligned}}$$



- a_2 must be significant for the curvilinear effect to be supported.

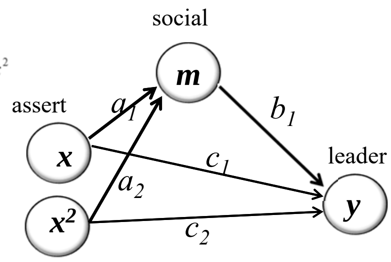
134

Mplus program

$$\boxed{\theta = (a_1 + 2a_2x)\, b_1}$$

$$\boxed{\begin{aligned} m &= a_0 + a_1x + a_2x^2 \\ y &= b_0 + b_1m \end{aligned}}$$

MODEL CONSTRAINT:
new (theta1 theta2 theta3);
new (predm1 predm2 predm3);
new (x1 x2 x3);
x1 = 3.9460; $x + \sigma$
x2 = 5.2275; $\bar{x}$
x3 = 6.5090; $x - \sigma$
predm1 = a0+a1*x1+a2*x1*x1;
predm2 = a0+a1*x2+a2*x2*x2; $m = a_0 + a_1x + a_2x^2$
predm3 = a0+a1*x3+a2*x3*x3;
theta1 = (a1+2*a2*x1)*b;
theta2 = (a1+2*a2*x2)*b; $\theta = (a_1 + 2a_2x)\, b_1$
theta3 = (a1+2*a2*x3)*b;
OUTPUT:
cinterval (bcbootstrap);



- The 3 Predm helps to show how the x→m relationship is curvilinear.
- The 3 theta helps to check whether the indirect effect x→m →y is curvilinear.

135

Generalization to …

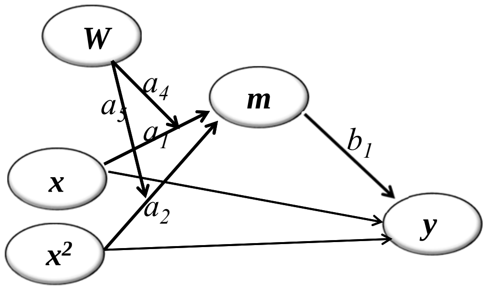
Curvilinear MoMe

Lin, B., Law, K., & Zhou, J. (2014). Why is underemployment related to creativity and OCB? A task crafting explanation of the curvilinear moderated relations. Academy of Management Meeting, Philadelphia, Aug 1-5.

136

Curvilinear mediation

$$m = a_0 + a_1 x + a_2 x^2 + a_3 w + a_4 wx + a_5 wx^2$$



$$\theta = \left(\left[a_1 + a_4 w\right] + 2\left[a_2 + a_5 w\right]x\right)b_1$$

Sample results

$$\theta = \left(\left[a_1 + a_4 w\right] + 2\left[a_2 + a_5 w\right]x\right)b_1$$

Since the moderated curvilinear mediation effect size depends on both w and x, one would need to tabulate both parameters to check the effect.

	W_{High}	W_{Low}	Δw
x_{High}	-.026	-.155	.129 [.044 , .212]
x_{Low}	-.129	-.577	.449 [.123 , .771]
Δx	.102 [.003 , .201]	.422 [.086 , .757]	-.320 [-.568 , -.069]

For different values of W (W_H vs. W_L), the curvilinear mediation effect is significantly different.

THE END